Workshop on Hyperbolic structures on 3-manifolds and large scale geometry of Teichmüller space

Warwick, 16–20 July 2007

Abstracts for the talks

Ken Bromberg (Utah)

Grafting, drilling and the ending lamination classification

We will outline a proof of the ending lamination conjecture whose starting point is Minksy's apriori bounds on the length of closed geodesics. This is joint work with J. Brock, R. Evans and J. Souto.

Dick Canary (Michigan)

Introductory Bumponomics: Topology of deformation spaces of hyperbolic 3-manifolds

In this talk, we discuss the space $AH(M)$ of all (marked) hyperbolic 3-manifolds homotopy equivalent to a fixed compact 3-manifold M (with boundary.)

The topology of the interior of $AH(M)$ is quite simple and has been well-understood since the 1970's. However, in the last decade it has become clear that the global topology of $AH(M)$ is quite complicated.

In this talk, we will survey the history and discuss recent joint work with Brock, Bromberg and Minsky, which shows, in many cases, that the topology is well-behaved at "most" points in the boundary of $AH(M)$.

Yitwah Cheung (San Francisco)

Slow divergence and Unique ergodicity

Unique ergodicity is generally implied by some form of recurrence. For example, an interval exchange T is uniquely ergodic if the endpoints of T^n are uniformly distributed infinitely often (Boshernitzan's criterion). There is a nearly equivalent formulation in terms of measured foliations. Masur proved that a measured foliation is uniquely ergodic if the associated Teichmüller geodesic ray is recurrent when projected to moduli space. The converse is not true in general–there are uniquely ergodic measured foliations associated to divergent Teichmüller geodesics. In this talk, I will discuss general conditions under which a slowly divergent Teichmüller geodesic is necessarily determined by a uniquely ergodic measured foliation. As an illustration of this circle of ideas, we give a new proof of Boshernitzan's criterion. This work is joint with Alex Eskin.

Moon Duchin (UC Davis)

Boundary phenomena in Teichmüller space

We discuss and relate various boundaries at infinity of Teichmüller space, the mapping class group, and the curve complex. In particular, we study the "stars at infinity" in the Thurston boundary of Teichmüller space (this part is joint work with Joseph Maher).

David Dumas (Brown)

Slicing, skinning, and grafting

We show that a Bers slice of quasifuchsian space is never algebraic, meaning that its Zariski closure has strictly larger dimension. It follows that the skinning map of a hyperbolizable 3-manifold with boundary is never constant. The proof uses grafting and the theory of complex projective structures on surfaces. This is joint work with Richard Kent.

Sergio Fenley (Florida State University)

Asymptotics of foliations and ideal boundaries of pseudo-Anosov flows

We consider pseudo-Anosov flows which are homotopically compatible (meaning that no closed orbit is freely homotopic to the inverse of another orbit). Using only the flow we produce a flow ideal sphere to the universal cover. We show that the action of the fundamental group G on the flow ideal sphere is a uniform convergence group. This implies that G is Gromov hyperbolic and that the action on the flow ideal sphere is conjugate to the action of G on its Gromov ideal boundary. This then implies that homotopically compatible pseudo-Anosov flows are quasigeodesic.

Finally this implies that if a foliation is R-covered or has one sided branching in an atoroidal three manifold, then the foliation satisfies the continuous extension property. This means that the leaves in the universal cover extend continuously to the sphere at infinity.

Koji Fujiwara (Tohoku)

Hyperbolicity, quasi-homomorphisms and stable commutator length

Let G be a group and G' its commutator subgroup. For an element g in G', we define its commutator length, $cl(g)$, to be the least number of commutators c_i to write g as a product $g = c_1 ... c_k$.

The stable commutator length of g, $scl(g)$, is defined by $\liminf_n cl(g^n)/n$.

I discuss the image $Im(scl)$ of the function $scl : G' \to R$. Gromov observed that if G has hyperbolicity, then it tends to happen that there is a positive lower bound on scl. I will talk about mappings class groups and the fundamental group of a non-positively curved Riemannian manifold of finite volume. I use a technique to construct quasi-homomorphisms on G, which Epstein and I found for word-hyperbolic groups.

François Guéritaud (Orsay)

Delaunay triangulations of convex cores

Epstein and Penner described a natural, "Delaunay" decomposition of finite-volume hyperbolic manifolds into polyhedra. When the volume becomes infinite, their construction may degenerate: non-contractible cells, failure of local finiteness... We will review some of these phenomena, whose understanding leads to a new proof of the Bending lamination theorem for punctured tori.

Ursula Hamentstädt (Bonn)

Dynamical properties of the Teichmüller flow

We relate the asymptotic growth rate of closed orbits of the Teichmüller flow in fixed compact subsets or in neighborhoods of infinity of the moduli space $\mathcal{Q}(S)$ of quadratic differentials to dynamical properties of the flow.

Yoshikata Kida (Bonn)

Orbit equivalence rigidity for ergodic actions of mapping classÊgroups

We discuss an ergodic, essentially free, measure-preserving action ofÊ mapping class groups on a standard probability space and present aÊ certain rigidity property for such an action. This rigidity propertyÊ states that the orbit structure of such an action remembers almostÊ all information on the acting group and how it acts. As anÊ application, we can determine all locally compact second countableÊ groups containing a lattice isomorphic to mapping class groups.

Sadayoshi Kojima (Tokyo Inst.Technology)

Comparison of hyperbolic volumes with other invariants

I would like to discuss a few interesting observations of my colleagues, E. Kin, M. Takasawa, and a student, H. Ohyama, obtained by their attempts of comparing the value of volumes of hyperbolic 3-manifolds with that of other invariants, such as dilatations of pseudo Anosov maps and special values of colored Jones polynomials.

Cyril Lecuire (Toulouse)

Thurston's cracked eggshell

Consider the set of faithful discrete representations (in the group of isometries of the 3-dimensional hyperbolic space) of a freely indecomposable Kleinian group. I will discuss the topology of this space when it is equipped with the strong topology. I will also investigate the action of the associated modular groups on this space.

Chris Leininger (Urbana)

The boundary of the curve complex

The boundary of the curve complex of a surface is connected, provided the surface has genus at least four, or has genus at least two and one or more punctures. This is joint work with Saul Schleimer.

Yair Minsky (Yale)

Coarse geometry of mapping class groups

Abstract TBA

Hossein Namazi (Princeton)

Splittings, hyperbolic geometry and models

A very important question about the geometry of hyperbolic manifolds is to construct the hyperbolic metric and describe it based on the topological information provided from a description of the 3-manifold. We will discuss an approach which relates combinatorial properties of a Heegaard splitting to important features of the hyperbolic structure on the 3-manifold. This is part of a joint work with Jeff Brock, Yair Minsky and Juan Souto.

Walter Neumann (Columbia)

Eigenvalue equivalence of hyperbolic manifolds

The talk has a subtitle: "Can you really hear the size of a drum?" I will describe joint work with McReynolds, Leininger, and Reid, on what the set of Laplace eigenvalues and set of primitive geodesic lengths tell one about a hyperbolic manifold.

Ken'ichi Ohshika (Osaka)

Divergence, exotic convergence and self-bumping in quasi-Fuchsian spaces

It is one of the important topics in the theory of Kleinian groups to determine the topological structure of deformation spaces of Kleinian groups. The first step of such a study would be to give a criterion for sequences in deformation spaces to converge or diverge. In this talk, we focus on the case of surface Kleinian groups and show a divergence theorem in a form much generalised as before. This should lead to understanding how quasi-Fuchsian groups can or cannot converge to a b-group, and when self-bumping can or cannot happen.

Kasra Rafi (Storrs, Connecticut)

Rigidity of the curve complex

We will show that if the boundary of a curve complex is connected then it is quasi-isometrically rigid. That is, every quasi-isometry from this curve complex to itself is bounded away from an isometry. This is joint work with Saul Schleimer.

Mary Rees (Liverpool)

Estimates on pleated surfaces

In a return to basics, some fundamental results on pleated surfaces will be reviewed. It will be shown how they can be adapted, especially in the absence of π_1-injectivity, for use in the proof of the general case of ELC.

Saul Schleimer (Warwick)

Covers and the curve complex

We provide the first non-trivial examples of quasi-isometric embeddings between curve complexes. These are induced either by puncturing a closed surface or via orbifold coverings. As a corollary, we give new quasi-isometric embeddings between mapping class groups. This is joint work with Kasra Rafi.

Ken Shackleton (Tokyo Inst.Technology)

On the coarse geometry of the Weil-Petersson metric

We begin by discussing joint work with Javier Aramayona and Hugo Parlier, studying the geometry of the pants graph of a surface after Hatcher and Thurston. Inspired by a recent theorem of Brock's, the aim of this work is to reinforce the pants graph as a good combinatorial model for the Weil-Petersson metric. We then discuss a question of Brock's and in two important cases prove, in outline, the existence of an algorithm for the computing of distances in the pants graph. This proof is outlined in a preliminary version of a proceedings article for the All Japan Topology Symposium, available at the URL `http://www.maths.soton.ac.uk/~kjs/Aizu.pdf`.

Teruhiko Soma (Tokyo Metropolitan)

Geometric limits of quasi-Fuchsian groups

We will determine the topological types of hyperbolic 3-manifolds $\mathbf{H}^3/G$ such that G is a geometric limit of an algebraically convergent sequence of quasi-Fuchsian groups. The proof is much cleared and simplified compared with previous ones by myself.

Juan Souto (Chicago)

Rank of the fundamental group and geometry of hyperbolic 3-manifolds

I will describe recent results towards the understanding what can be said about the topology and geometry of a closed hyperbolic 3-manifold once the rank of its fundamental group is know.

Scott Wolpert (Maryland)

The $CAT(0)$ geometry of Teichmüller space

We provide an overview of the continuing development of the Weil-Petersson metric geometry of the augmented Teichmüller space $\overline{\mathcal{T}}$. An invariant of a marked hyperbolic structure is the length ℓ_α of the geodesic α in a free homotopy class. A basic feature of Teichmüller theory is the interplay of two-dimensional hyperbolic geometry, WP geometry and the behavior of geodesic-length functions. We present

- an intrinsic expansion for the WP metric and Levi-Civita connection,
- a characterization of finite length WP geodesics,
- a description of the Alexandrov tangent cones for the augmented Teichmüller space.

Applications include existence statements for combinatorial harmonic maps into $\overline{\mathcal{T}}$ and for fixed points of group actions.

A WP quick-fact sheet is available at `http://arxiv.org/abs/0705.1105`