David Epstein's 70th Birthday Celebration

Warwick, 13–14 July 2007

Abstracts for the talks

Brian Bowditch (Southampton)

Atoroidal surface-by-surface groups

There are many open questions regarding the geometry of surface bundles over surfaces. For example it is unknown if there is any surface-by-surface group with no free abelian subgroup of rank two. One can however show that there are only finitely many isomorphism classes of such groups for given base and fibre genera. The proof uses ideas from the geometry of hyperbolic 3-manifolds, in particular in relation to the Ending Lamination Conjecture.

Jim W. Cannon (Brigham Young)

Kleinian groups and rational maps as dynamical systems

The action of a Gromov hyperbolic group on its space at infinity (viewed as its limit set or Julia set) can be captured by a single function from a (non-Hausdorff) "manifold" to itself. (The "manifold", or perhaps more accurately "orbifold", is locally modelled on the space at infinity, whatever the topology of that space at infinity may be.) This point of view allows us to view the study of Kleinian groups and rational maps as a single subject, namely the dynamical study of an iterated map on a manifold.

We describe some of the unifying theorems, examples, and open questions.

Daryl Cooper (UCSB)

Projective geometry and low dimesional topology

We will discuss various aspects of projective (and related) structures on surfaces and 3-manifolds.

David Gabai (Princeton)

Volumes of hyperbolic 3-manifolds

We outline a proof that the Weeks manifold is the lowest volume closed orientable hyperbolic 3-manifold.

Morris Hirsch (Wisconsin)

Nonlinear representations of Lie algebras on manifolds

A C^r *action* α of a finite dimensional Lie algebra $\mathfrak{g}$ on a manifold M is a linear map from $\mathfrak{g}$ to the space of C^r vector fields on M that commutes with brackets if $r \geq 1$, or if $r = 0$ it means a local action on M of some Lie group whose Lie algebra is $\mathfrak{g}$. If $\dim M = n$ then α is an *n-action.*

The *Epstein-Thurston Theorem* says that if $\mathfrak{g}$ is solvable (respectively, nilpotent) and has an effective n-action then $\mathfrak{g}$ has derived length $\ell(g) \leq n+1$ (resp., $\leq n$). But this is far from sufficient for the existence of effective n-actions:

Theorem For every $n \geq 2$ there is a nilpotent $\mathfrak{g}$ with $\ell(\mathfrak{g}) = 2$ that has an effective affine action on $\mathbf{R}^{n+1}$, but every n-action annihilates the center.

Generalized Turiel Theorem If a compact manifold N^{2k} admits an effective real analytic action β of the direct product of k nonsupersoluble algebras, then $\chi(N^{2k}) \geq \#\text{Fix}(\beta) \geq 0$. But this is false for C^∞ actions.

Spectral Rigidity Theorem Assume $X \in \mathfrak{g}$ and $\mathsf{ad}X$ has n eigenvalues that are linearly independent over the rationals. Let α be an effective analytic action of $\mathfrak{g}$ on M^n and assume X^α vanishes at $p \in M^n$. Given a neighborhood U of p there is a neighborhood $\mathcal{N}$ of α in the space of C^1 actions of $\mathfrak{g}$ on M^n, with the following property: If $\beta \in \mathcal{N}$ then X^β vanishes at a unique $q \in U$, and $dX^\beta(q)$ has the same spectrum as $dX^\alpha(p)$.

Al Marden (Minneapolis)

The view from above

A survey of work over the past 20 years successively unraveling the geometric relationship between a relative boundary component of a hyperbolic convex hull and the region it faces "at infinity" in the 2-sphere.

The talk is based on work of David Epstein, Vlad Markovic, and the speaker that originates in a remarkable discovery of Sullivan and Thurston.

Dusa McDuff (Stony Brook)

The structure of groups of symplectomorphisms

This will be about quasimorphisms (which are homomorphisms up to a bounded error) and their consequences for the behaviour of symplectic transformations and will mostly describe work by Polterovich and his collaborators. This is indirectly related to David's work on the simplicity of groups of homeomorphisms.

Greg McShane (Toulouse)

Dynamics of the modular group on the character variety

(Joint with Tan Ser Peow NUS, Singapore.)

The Teichmüller space of a surface of negative Euler characteristic comes with a discontinuous action of the mapping class group. The Teichmüller space embeds in the character variety, that is, the space of representations of the fundamental group of the surface into $\mathrm{SL}(2, \mathbf{C})$ up to conjugation. The action of the mapping class group extends to the character variety in a natural way.

Bill Goldman studied the restriction of this action to the real part of the character variety for the holed torus. He obtained a complete description of the dynamics: fixed points, recurrent part, dissipative part, wandering domains etc. Subsequently, with his student Stanchev, Goldman analysed the action on character variety of the non-orientable surfaces (Klein bottle, punctured Moebius band).

We present the work of Goldman and Stantchev and explain how to complete their program.

John Milnor (Stony Brook)

Modeling evolution

Mathematical models, even when highly simplified, may help us to understand the way in which the gene pool of a species changes with time. The talk will describe a model based on spherical geometry.

Rich Schwartz (Brown)

Unbounded orbits for outer billiards

Outer billiards is a simple dynamical system based on a convex shape in the plane. B.H. Neumann introduced this system in the 1950s and J. Moser popularized it in the 1970s as a toy model for celestial mechanics. All along, one of the central questions has been whether or not one can find a shape for which the system has an unbounded orbit. In my talk I will explain my solution to this problem. It turns out that outer billiards relative to almost every kite has some unbounded orbits. (A kite is a kite-shaped symmetric quadilateral.) My proof relates outer billiards on kites to self-similar tilings, arithmetic-type dynamics, Diophantine approximation, and polytope exchange maps.

Zlil Sela (Hebrew University)

Some applications of low dimensional topology in model theory

We combine the techniques that were develpoed in solving the Tarski problem on the elementary theory of free groups with concepts from low dimensional topology, to answer some basic model theoretic questions on the first order theory of free (and hyperbolic) groups.