Universal Features in Turbulence:
From Quantum to Cosmological Scales
Warwick-Dec. 6, 2005

Producing and Probing Quantum Turbulence

Gary Ihas
Lancaster University

and

University of Florida

Funding: EPSRC and Research Corporation
Large cast of contributors:
G. Labbe, S-c. Liu, R. Adjimambetov, M. Padron, W.F. Vinen,
P.V.E. McClintock, D. Charalambous, P.C. Hendry, V. Mitin

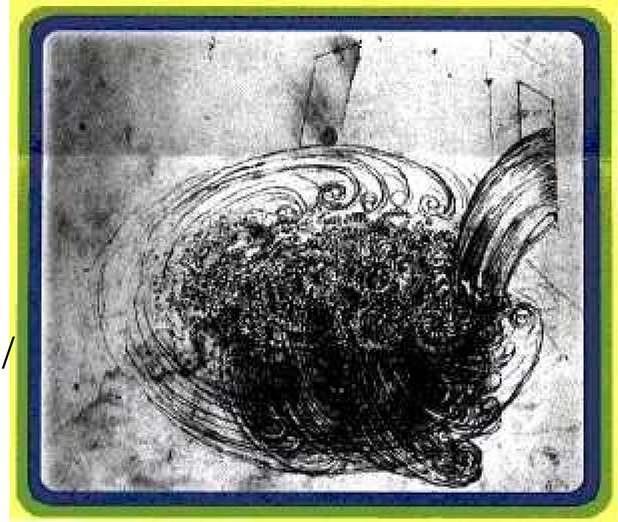
Our Problem

- Study of turbulence in a classical fluid based on many detailed experimental observations..

  - simple direct visualization of the flow (Leonardo da Vinci onwards)

a free water jet issuing from a square hole into a pool →

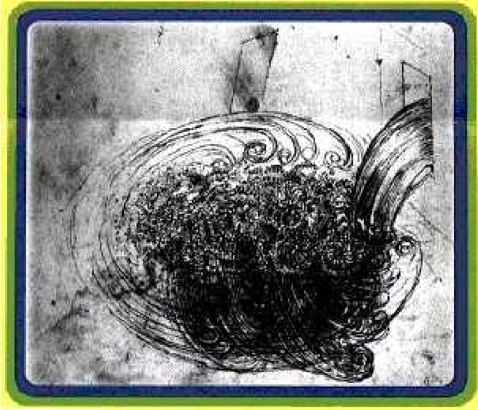
http://www.emicronano.com/efluids/gallery/leonard_vortices.html



  - measurement of forces and pressure gradients
  - measurement of velocity fields (hot wires; laser Doppler; PIV)
  - measurement of correlation functions, energy spectra, etc.

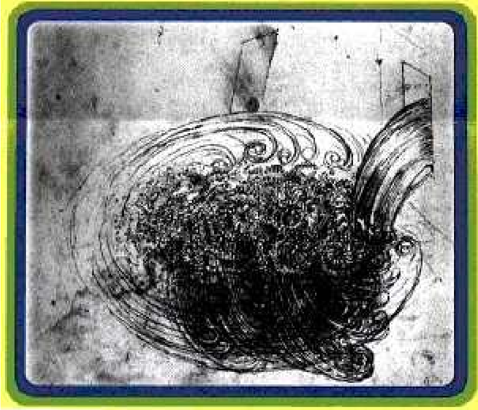
- **In contrast**, direct observations of quantum turbulence very limited

Leonardo da Vinci describing the flow



“Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion…
The small eddies are almost numberless, and large things are rotated only by large eddies and not by small ones, and small things are turned by both small eddies and large.”

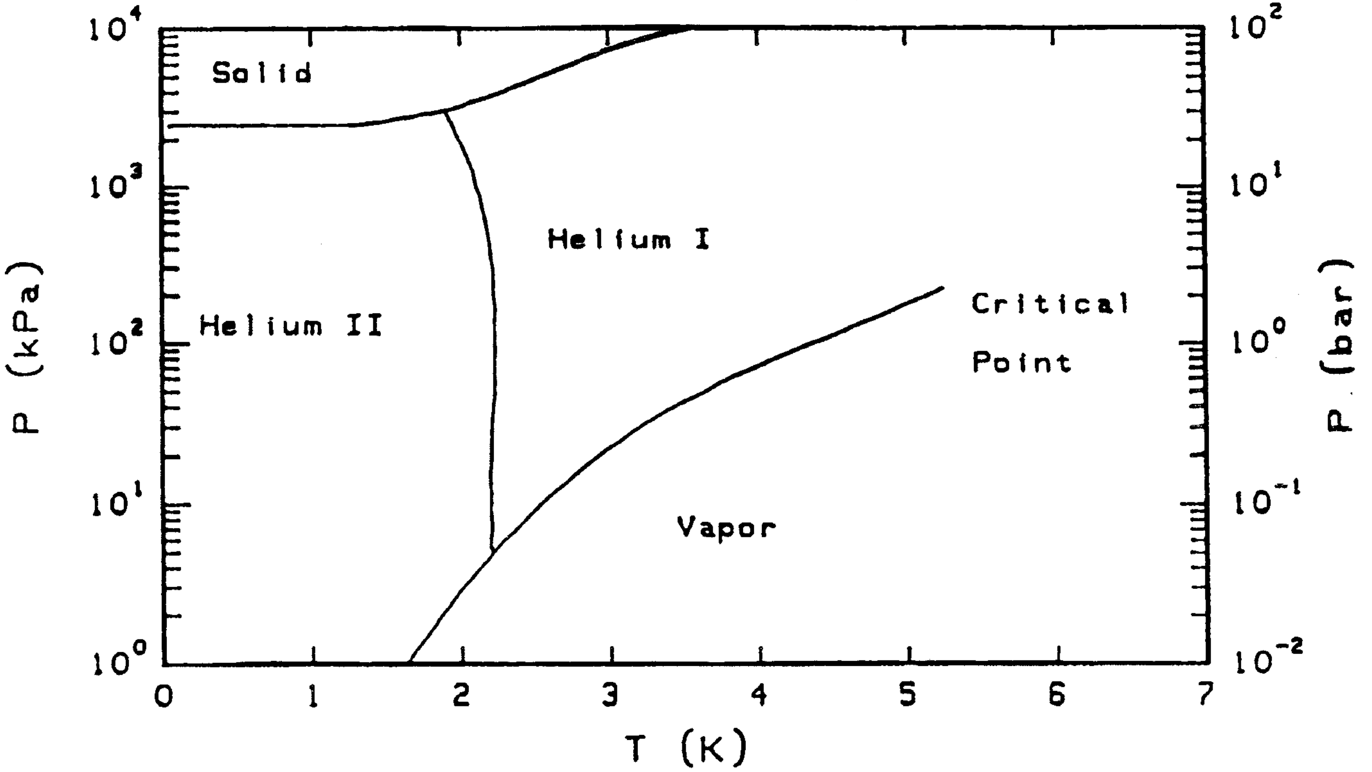
Leonardo da Vinci
Our Friend



“No knowledge can be certain, if it is not based upon mathematics or upon some other knowledge which is itself based upon the mathematical sciences.”

“Instrumental or mechanical science is the noblest and above all others, the most useful.”

^{4}He phase diagram



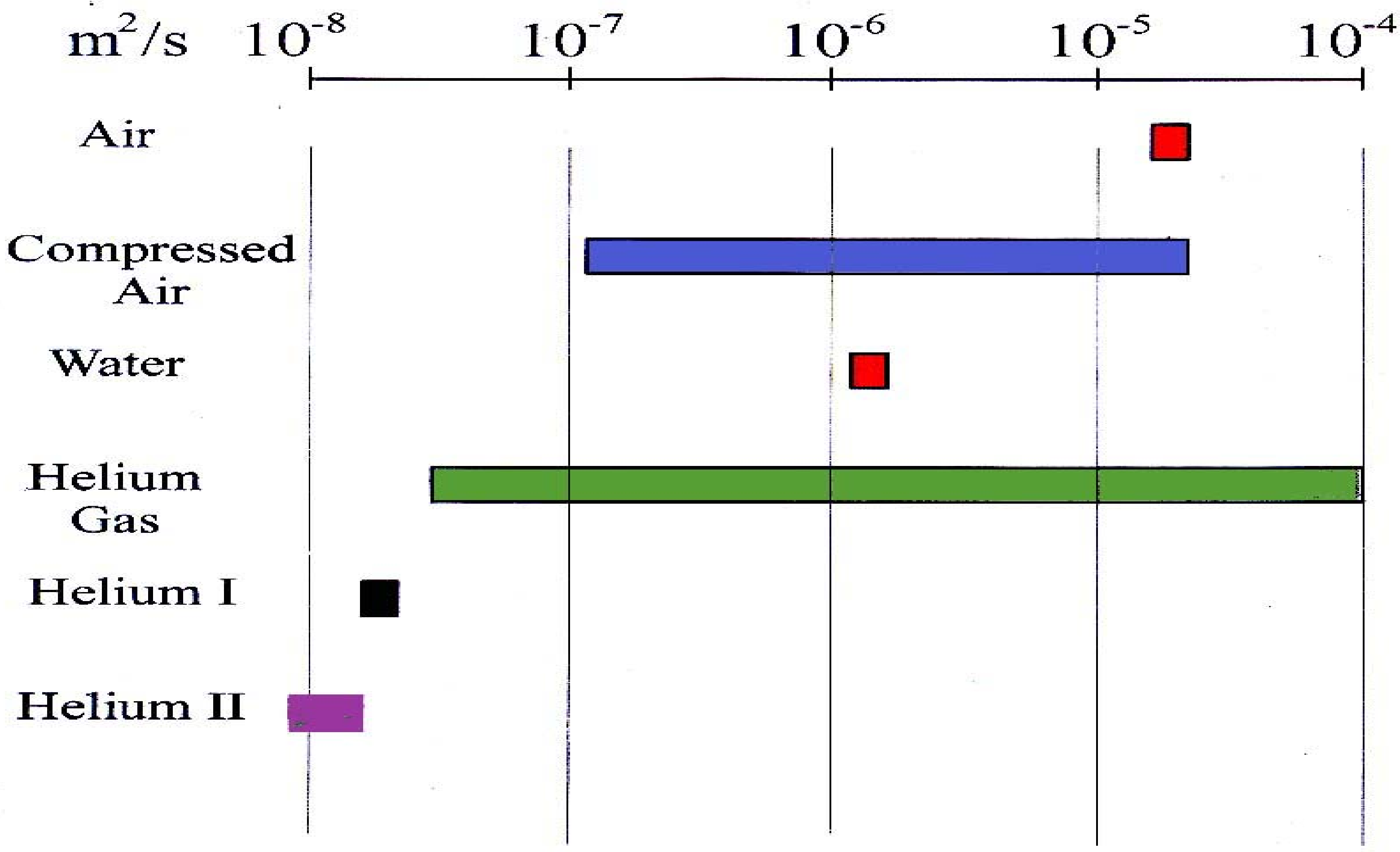
superfluidity breaks down due to production of quantized vortices

$$\psi(\mathrm{r}) = \psi_0 \mathbf{e}^{\mathrm{iS(r)}} \longrightarrow \mathbf{v}_\mathrm{s} = \frac{\hbar}{\mathrm{m}_4}\nabla\mathbf{S} \longrightarrow \nabla \times \mathbf{v}_\mathrm{s} = 0$$

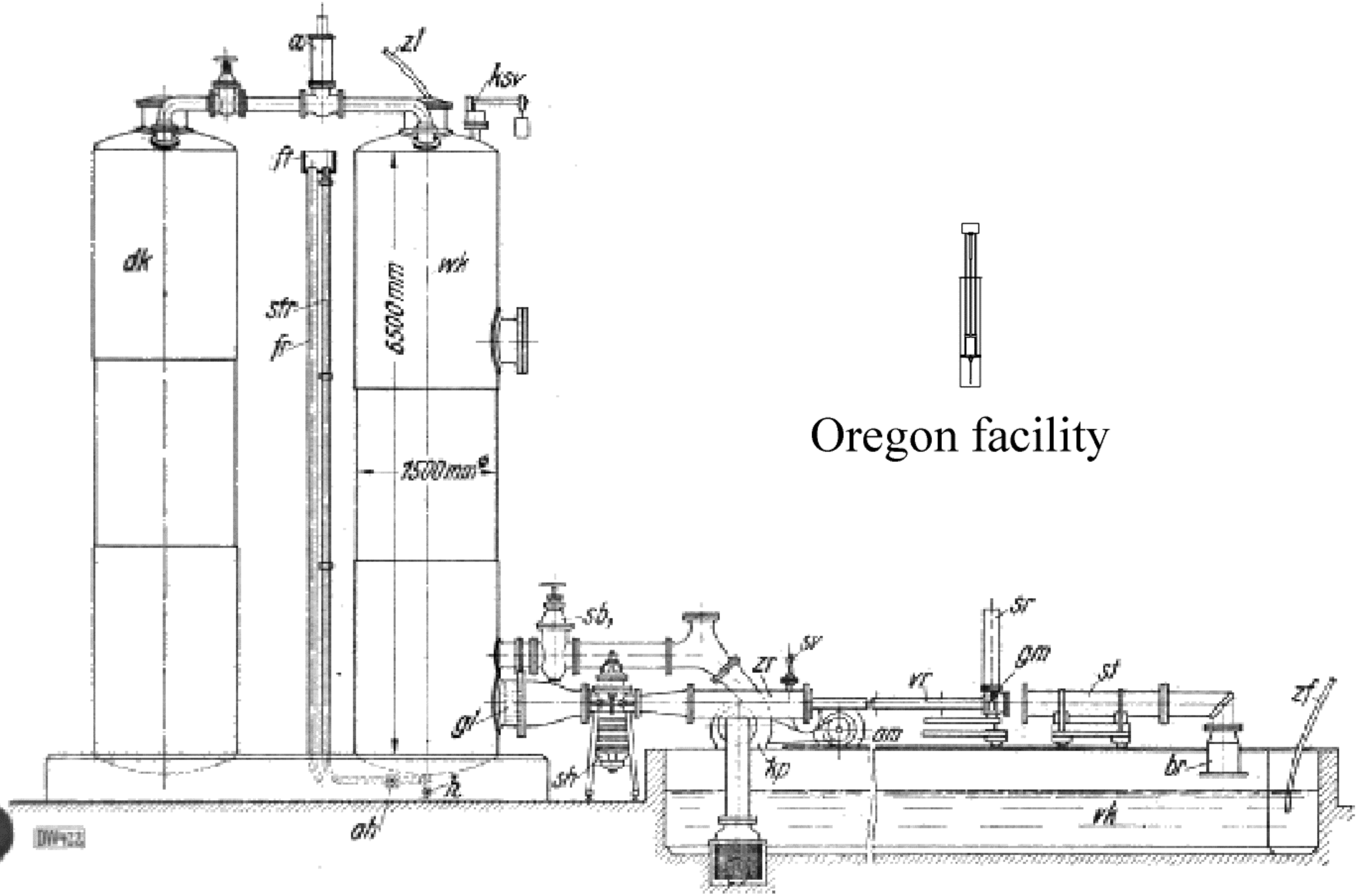
Superfluid is *irrotational*, but can have finite circulation about a line singularity (hollow vortex core):

$$\kappa = \oint \mathbf{v}_\mathrm{s} \cdot \mathrm{d}\mathbf{l} = \frac{\hbar}{\mathrm{m}_4}\Delta\mathbf{S} = \mathbf{n}\left(\frac{\mathrm{h}}{\mathrm{m}_4}\right)$$

$$\mathrm{Re} = \frac{UL}{\nu}$$



Kinematic Viscosity of Fluids for Turbulence Research



Nikuradze’s Water facility

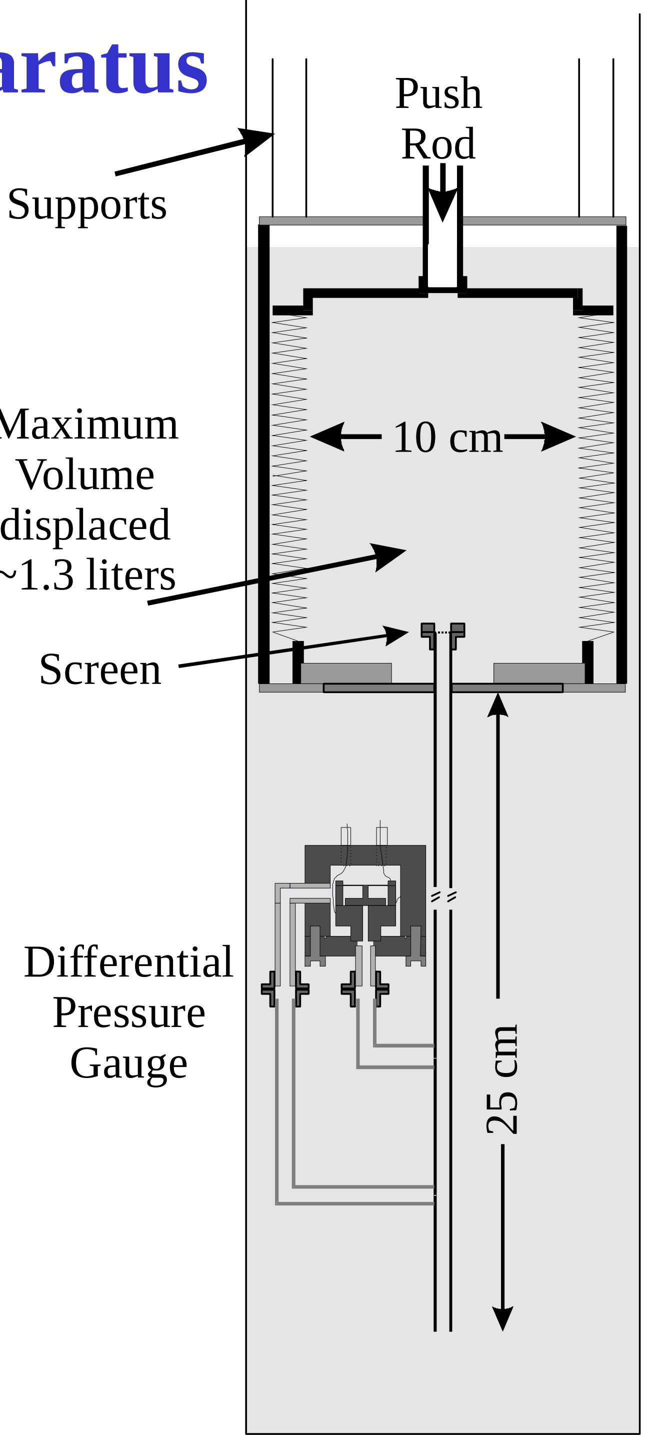
Oregon facility

Oregon Pipe Flow Apparatus

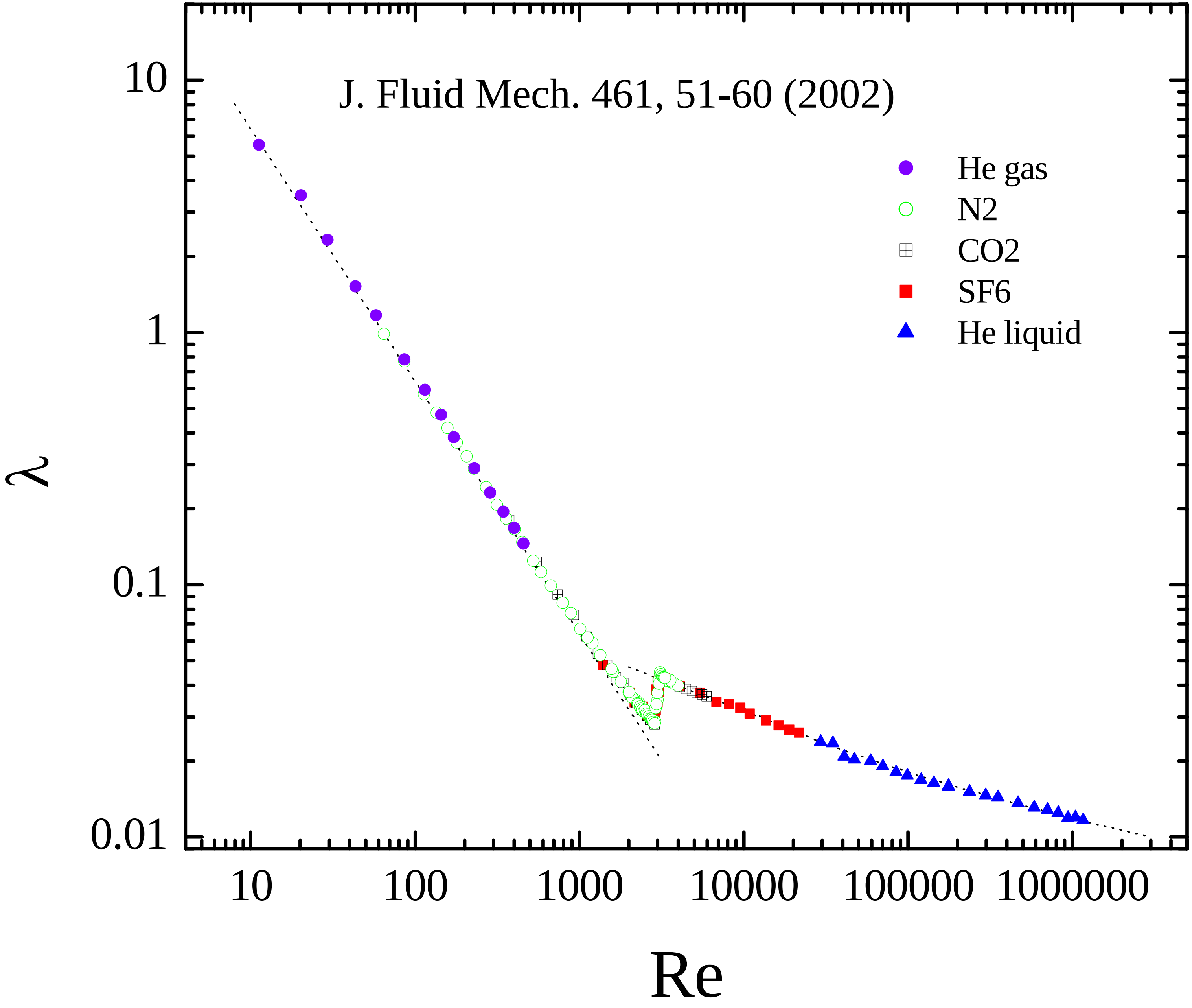
Developed strain-type (similar to UF "Straty-Adams" gauge) *in situ* capacitive probe with resolution of 1 Pa

Requires at least 15 J for 1 measurement

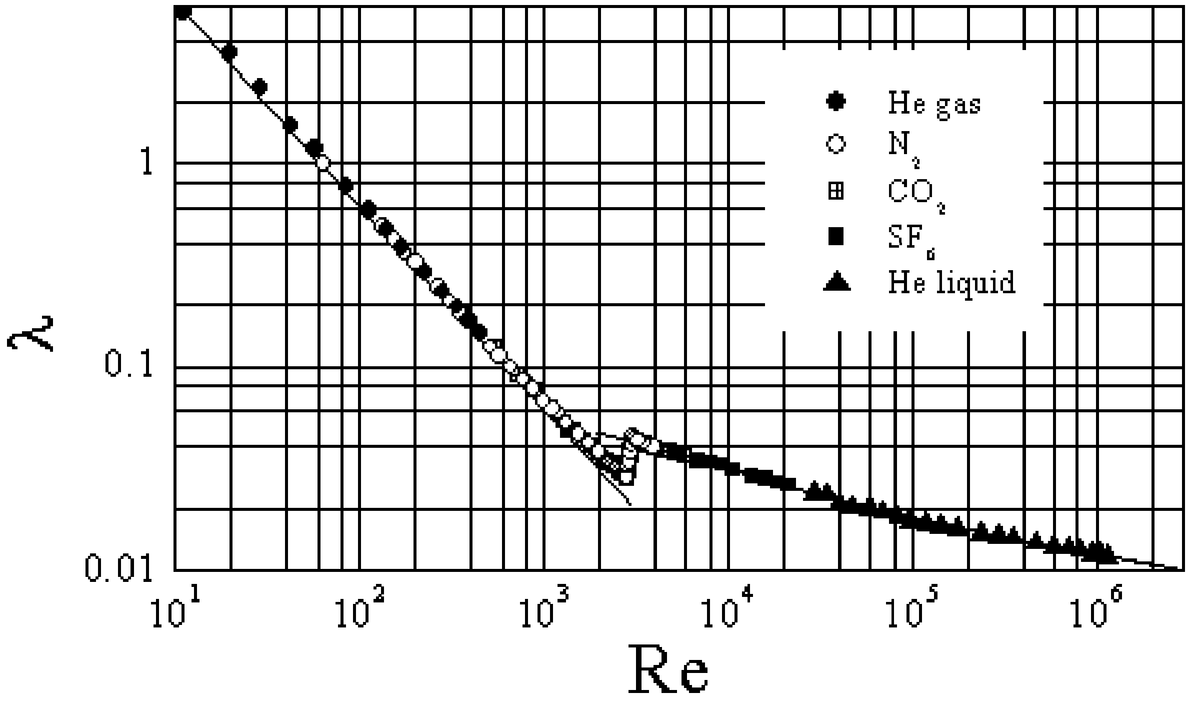
capacitor plate moves 0.01 Å



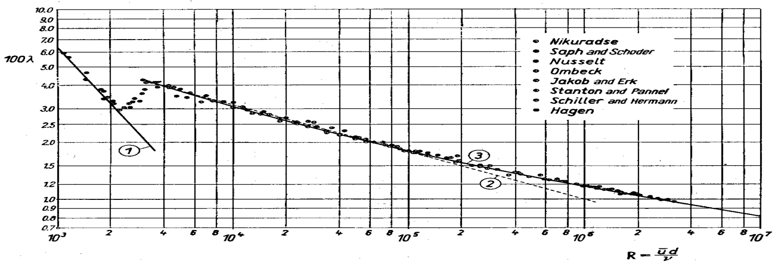
Friction Coefficients for 5 Fluids



Results similar to Classical Fluids



From our measurements at Oregon

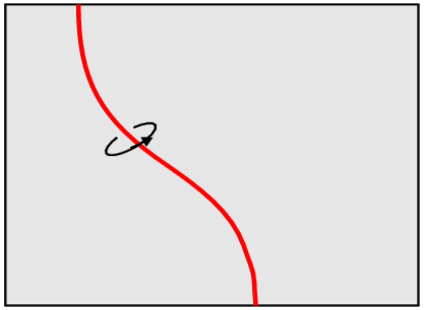


From Schlichting’s book on Boundary layer theory

Simple superfluids I -- a la Vinen

- ☐ Superfluids (^{4}He; ^{3}He-B; cold atoms) exhibit
  - Two fluid behaviour: a viscous normal component + an "inviscid" superfluid component. Normal component disappears at lowest temps.
  - Quantization of rotational motion in the superfluid component.

  (Consequences of Bose or BCS condensation.)



- ☐ Quantization of rotational motion: curl $\mathbf{v}_s = 0$, except on quantized vortex lines, each with one quantum of circulation

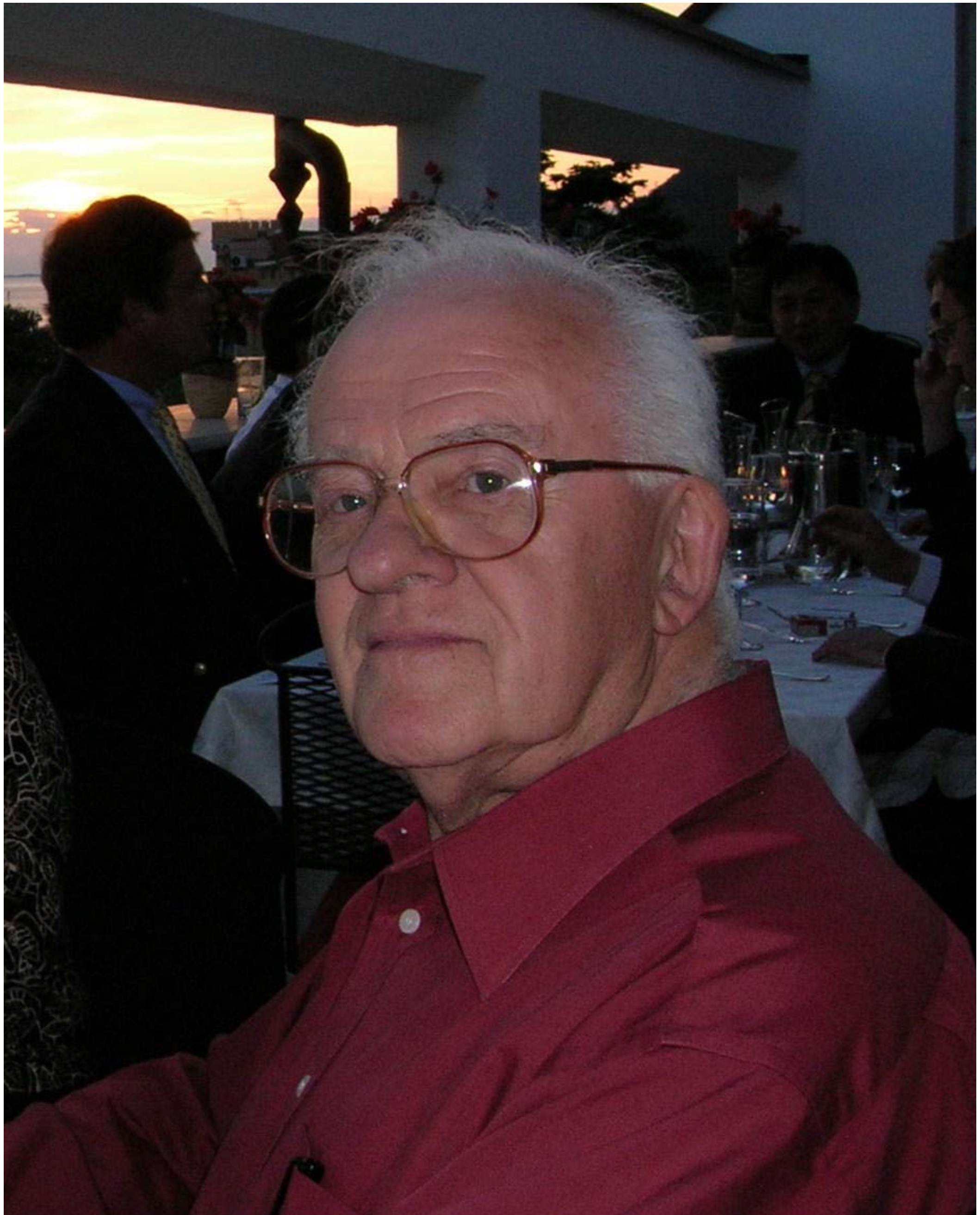
$$\kappa = \oint \mathbf{v}_s \cdot d\mathbf{r} = h/m_4 \text{ or } h/2m_3$$

round a core of radius equal to the coherence length ξ

(ξ ~0.05 nm for ^{4}He; ~80nm for ^{3}He-B; larger for Bose gases).

- ☐ Kinematic viscosity of normal fluid: ^{4}He very small; ^{3}He-B very large. Turbulence in normal fluid? ^{4}He: **YES**; ^{3}He-B: **NO**.

50th Anniversary of First Direct Detection of Quantized Vorticity

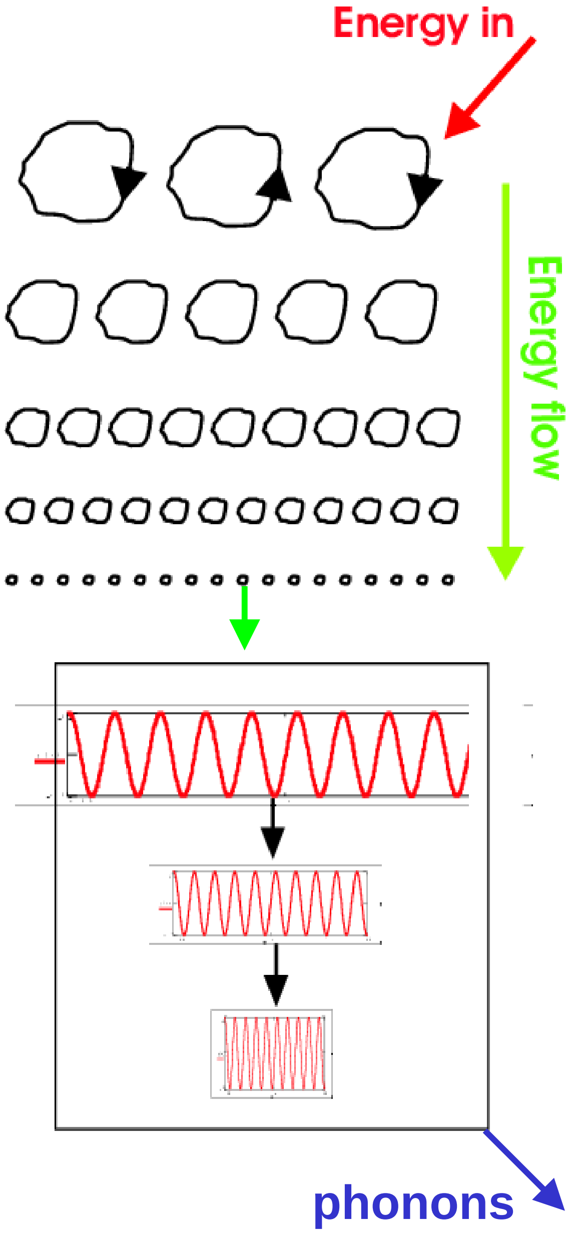


Observations of quantum turbulence

1. Ion trapping -- Lancaster and Manchester
2. Second sound -- Prague, Grenoble and Oregon
3. Rotating spheres-drag--Regensberg
4. PIV -- Maryland (Yale) and FSU
5. Convection -- ICTP
6. Calorimetry --in progress at Lancaster and Florida
7. Excimers -- proposed by McKinley and Vinen
8. NMR- in ^{3}He --Helsinki
9. Quasiparticles in ^{3}He --Lancaster
10. Shadow graphs -- Manchester

A good start, but no measurements over the range of scales needed and little or no direct visualization!

Decay of homogeneous isotropic turbulence at $T = 0$



As produced by a moving grid in ^{4}He.

(In wake of grid energy input is zero)

Classical Richardson cascade on scales greater than line spacing ℓ.

Energy = E_{class}

energy flow $\varepsilon = \frac{dE_{class}}{dt}$

Kelvin wave cascade on scales less than ℓ . Energy $= E_{Kelvin}$

energy flow $= \varepsilon + \frac{dE_{Kelvin}}{dt}$

Phonons

Neglect direct phonon generation during reconnections

Decay of homogeneous isotropic turbulence at $T = 0$ (cond)

Richardson cascade → $E_{class}(k) = C\varepsilon^{2/3}k^{-5/3}$

Kolmogorov spectrum

For decaying turbulence (no input of energy), we find (per unit mass)

$$E_{class} = \int_{d^{-1}}^{\ell^{-1}} E_{class}(k)dk = \frac{27}{2}C^3d^2(t+t_0)^{-2}$$

and

$$\varepsilon = 27C^3d^2(t+t_0)^{-3}$$

d = size of largest eddies, assumed constant

We can also write

$$E_{class} = \frac{3}{2}\beta\left(\frac{\kappa}{\ell}\right)^2\left(\frac{d}{2\pi\ell}\right)^{2/3}$$

$\kappa = h/m$ $\beta \approx 0.25$

Ensures that classical velocities join continuously to quantum velocities at $k = \ell^{-1}$

$$\ell = (2\pi)^{-1/4}\beta^{3/8}C^{-9/8}\kappa^{3/4}d^{-1/2}(t+t_0)^{3/4}$$

Kelvin-wave cascade → $E_{Kelvin}(\tilde{k}) \approx A\kappa^2\ell^{-2}\tilde{k}^{-1}$

Approximately independent of ε

$$E_{Kelvin} = \int_{\ell^{-1}}^{\tilde{k}_c} E_{Kelvin}(\tilde{k})d\tilde{k} \approx A\kappa^2\ell^{-2}\ln(\tilde{k}_c\ell)$$

$$\frac{dE_{Kelvin}}{dt} = A\kappa^2\ell^{-3}\left(1+\ln(\tilde{k}_c\ell)\right)\frac{d\ell}{dt}$$

$\tilde{k}_c$ = cut-off due to phonon emission (~ 2 nm^{-1})

Decay of homogeneous isotropic turbulence at *T* = 0 (cond)

Therefore total rate of flow of energy into phonons is

$$27C^3d^2(t+t_0)^{-3}+\left(\frac{9\pi}{8}\right)^{1/2}AC^{9/4}\beta^{-3/4}\kappa^{1/2}d\left(1+\ln\left(\tilde{k}_c\ell\right)\right)(t+t_0)^{-5/2}$$

Note different dependences on time in two terms.

Ratio of the two terms

$$\frac{\text{Kelvin}}{\text{Classical}}=\frac{3}{54}(2\pi)^{1/2}AC^{-3/4}\beta^{-3/4}\kappa^{1/2}d^{-1}\left(1+\ln\left(\tilde{k}_c\ell\right)\right)(t+t_0)^{1/2}$$

Typically

$$\frac{\text{Kelvin}}{\text{Classical}}=0.2(t+t_0)^{1/2}$$

so that typically the two terms are of comparable magnitude

So probably calorimetric measurements can provide information relevant to both the classical Richardson cascade and the Kelvin-wave cascade.

Probe requirements

- Length scales: wide range of scales from the size of the flow obstacle or channel giving rise to the turbulence to the (small) scale on which dissipation occurs.
  - E.g. turbulence in ^{4}He above 1K has energy-containing eddies of 1 cm and characteristic velocity 1 cm s^{-1}. Below 1K Kelvin wave cascade (Vinen) to dissipate energy may take smallest scale to 10 nm.
- Time scales: ranges from 1 s to a few milliseconds.
- Velocity correlation functions: play an important role in classical turbulence (structure functions). We could derive energy spectra from them and look for deviations from Kolmogorov scaling (higher-order structure functions).
- Do not underestimate the importance of visualizing the flow.

Second Sound and NMR

- Second sound, used successfully to measure vortex line density in ^{4}He, does not propagate below 1 K in ^{4}He or in superfluid ^{3}He.

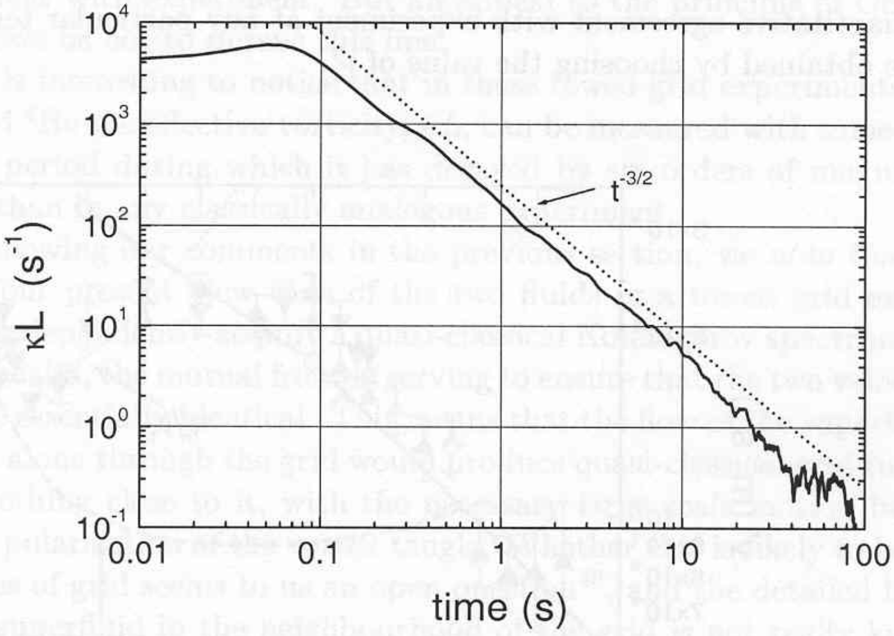
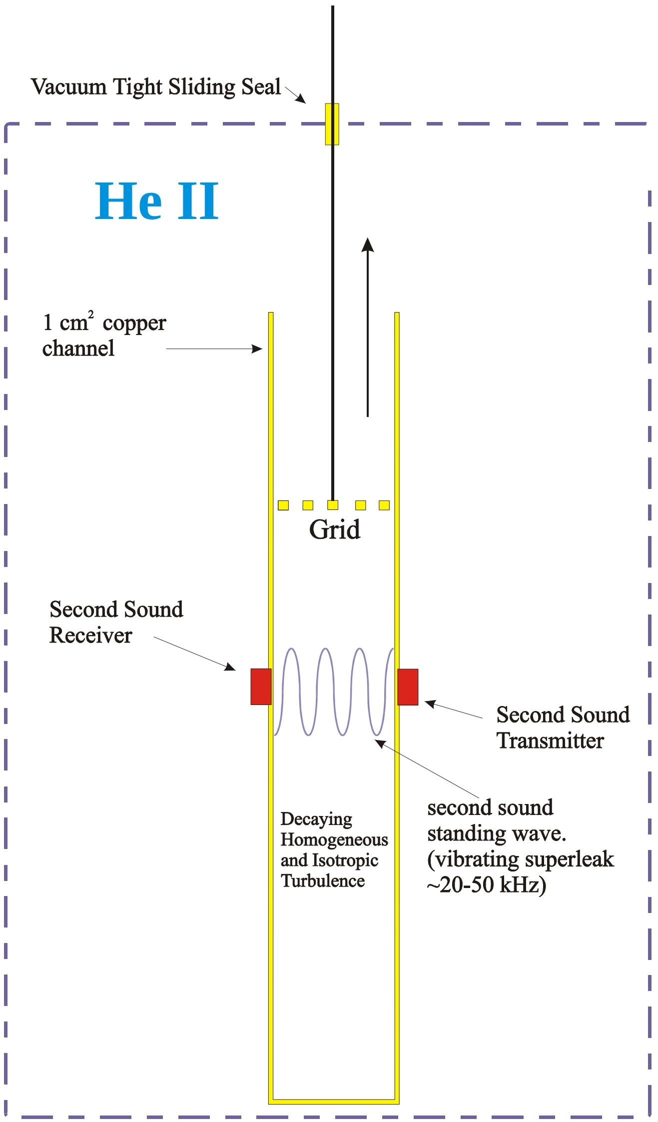


Fig. 7. The observed decay of vortex line density behind a towed grid. From thesis of S.R. Stalp (University of Oregon).

- However, NMR signals can be used to measure vortex densities in ^{3}He, with very high sensitivity.

Grid Turbulence



Vacuum Tight Sliding Seal

He II

1 cm² copper channel

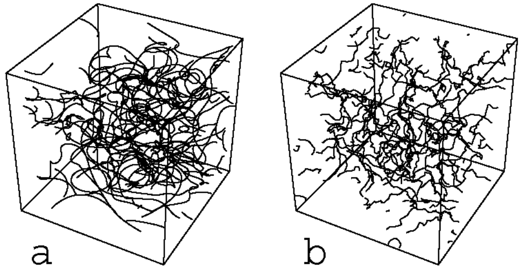
Grid

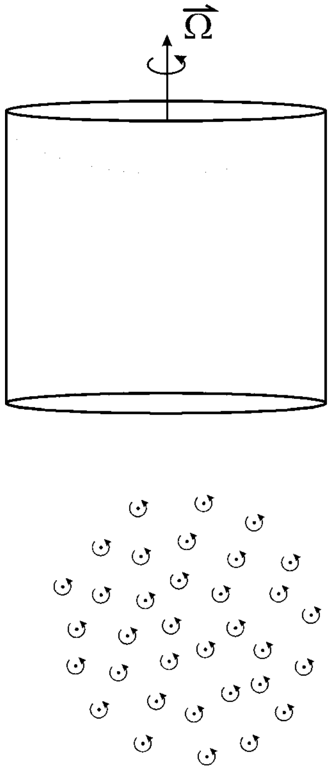
Second Sound Receiver

Second Sound Transmitter

Decaying Homogeneous and Isotropic Turbulence

second sound standing wave. (vibrating superleak ~20-50 kHz)





quantum of circulation:

$\kappa = h/m_4 \sim 10^{-3} \mathrm{cm}^2\ \mathrm{s}^{-1}$ (n=1)

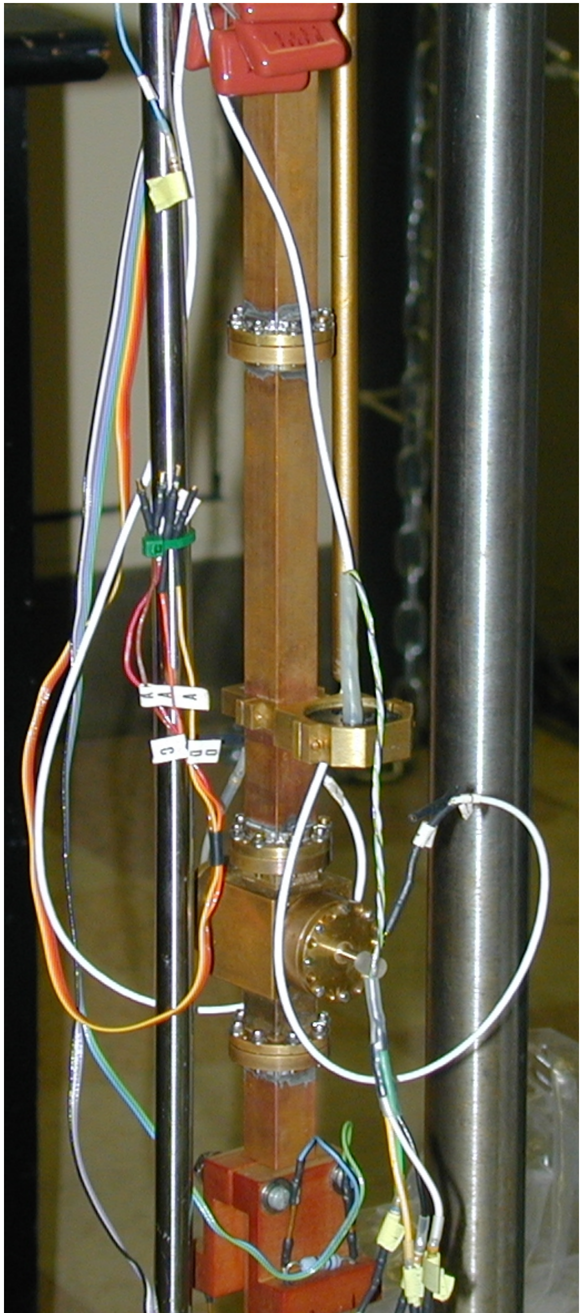
L = length of quantized vortex line per unit volume.

$\omega_s = \kappa L$ = total rms superfluid vorticity

$\ell = L^{-1/2}$ = average inter-vortex line spacing.

$$E_v = \frac{\rho_s \kappa^2}{4\pi} ln\left(\frac{r_{eff}}{a_0}\right)$$ (~10^{-7}erg cm^{-1}, $a_0 \sim 0.1$ nm, $r_{eff} \sim \ell$)

Stalp Pulled Grid Second Sound Apparatus



Apparatus size and mesh Reynolds numbers R_M in a few grid turbulence experiments

Source	test section	max R_M (million)
Kistler & Vrebalovich (1966) (air at 4 atmospheres)	2.6 m × 3.5 m	2.3
Comte-Bellot & Corrsin (1971) (atmospheric air)	1 m × 1.3 m	0.3
Oregon towed grid (He II)	1 cm × 1 cm	0.5
Yale towed grid (He I)	5 cm × 5 cm	0.8

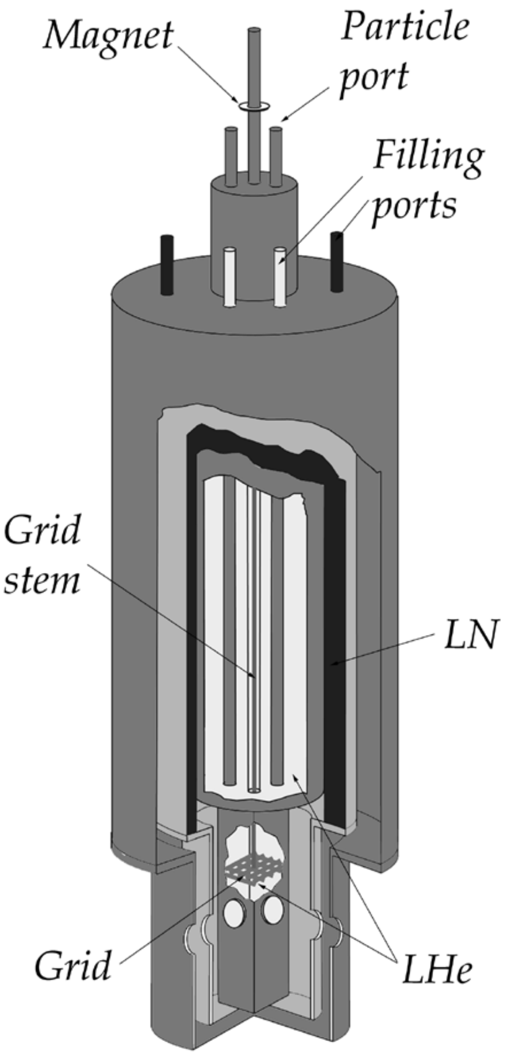
PIV Technique

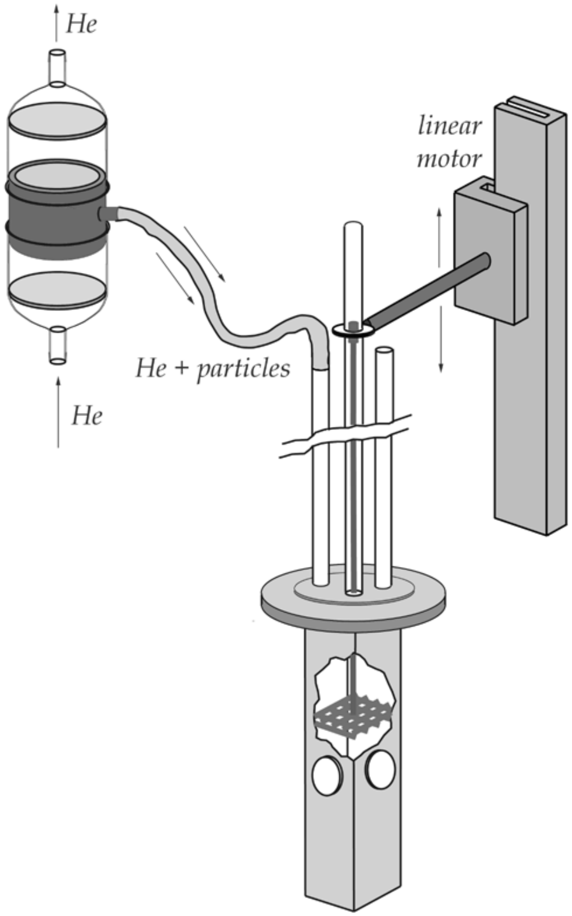
Particle Image Velocimetry

Sequential snap shots in time are compared to follow trajectories of tracer particles immersed in flowing fluid

http://www.grc.nasa.gov/WWW/OptInstr/piv/pivdemo.htm

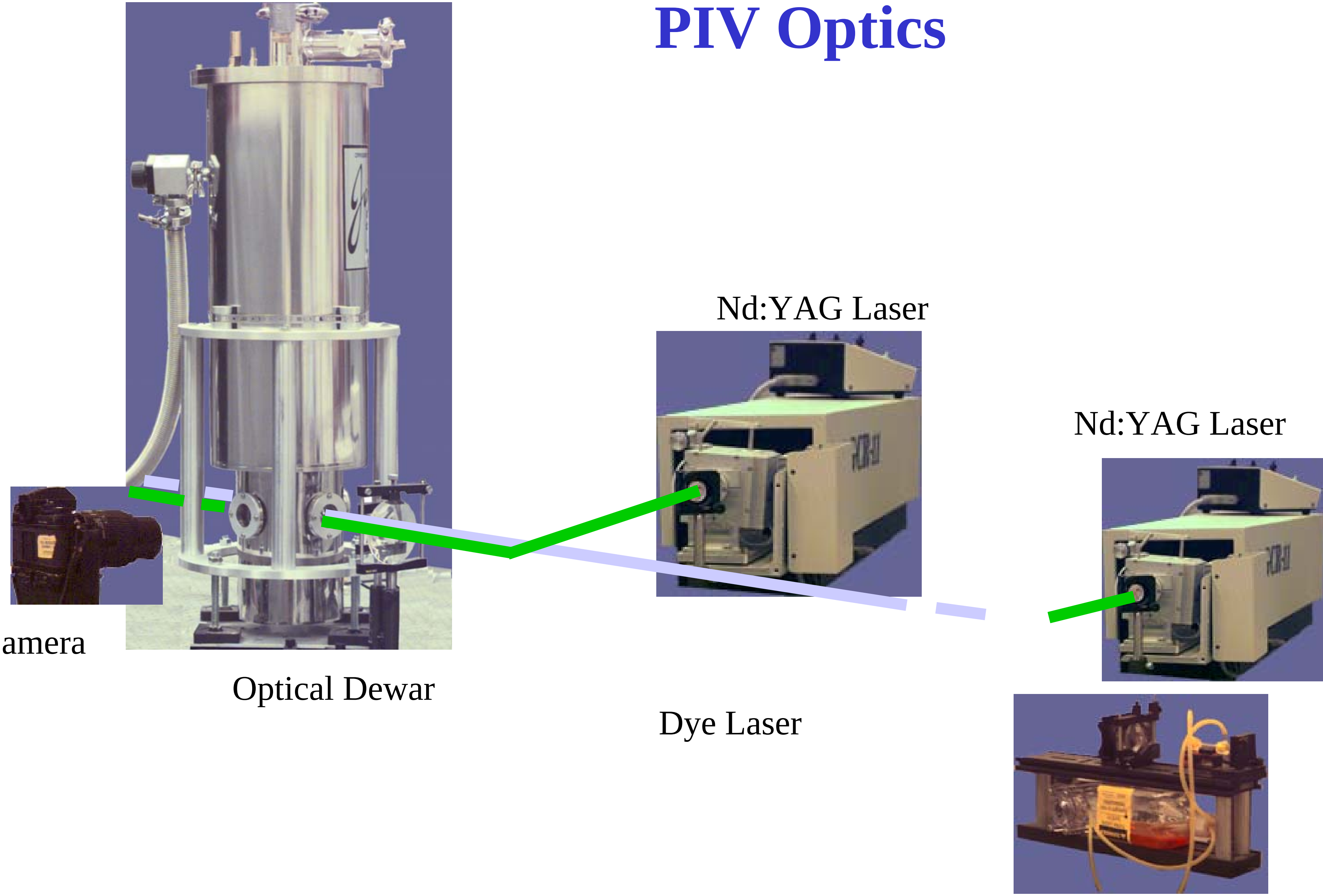
PIV Pulled Grid Apparatus





C.M. White, A.N. Karpetis and K.R. Sreenivasan *J. Fluid Mech.* **452**, 189-197 (2001); C.M. White, Ph.D. thesis, Yale U. (2001)

PIV Optics



Nd:YAG Laser

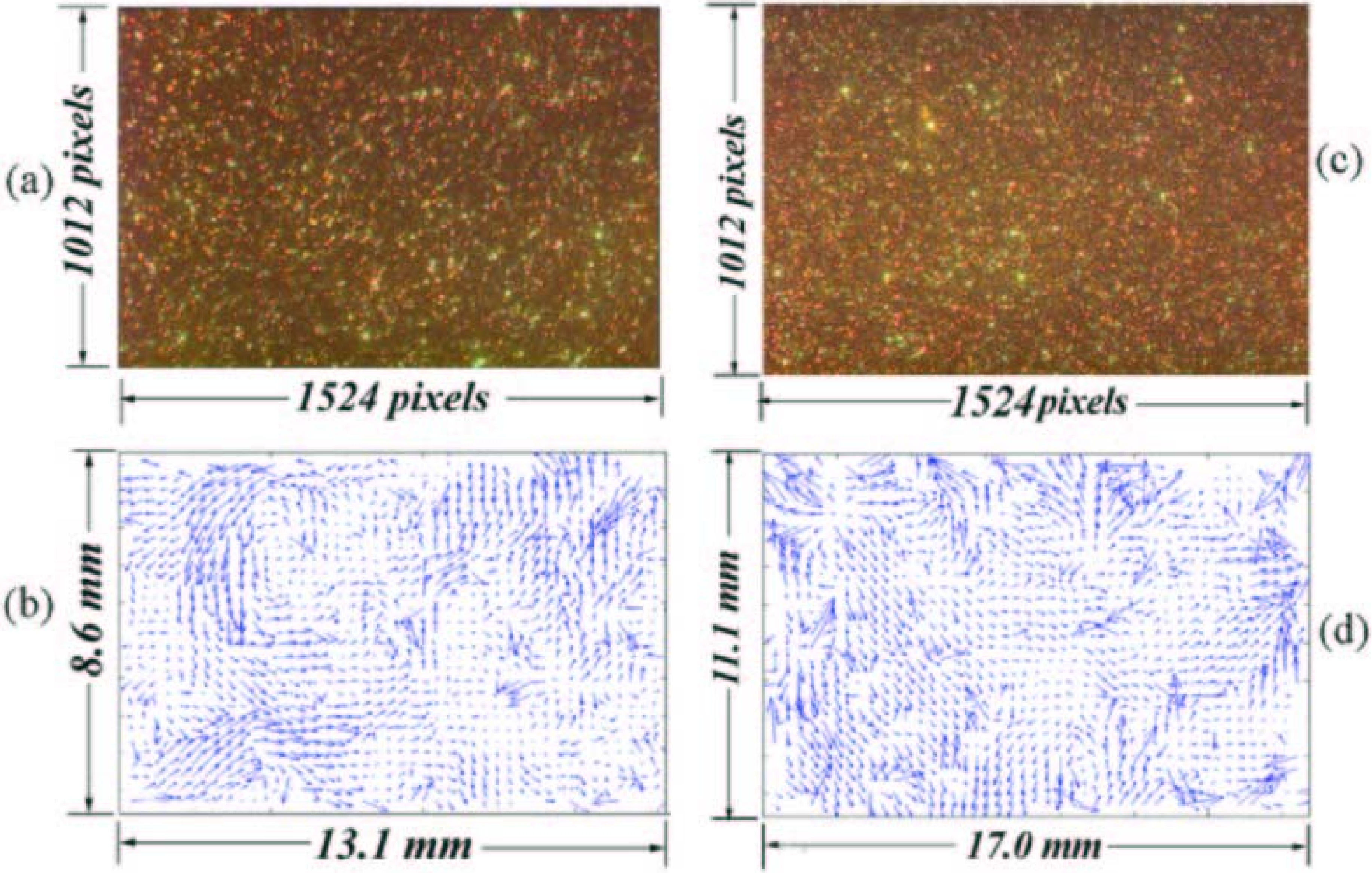
Nd:YAG Laser

Camera

Optical Dewar

Dye Laser

Flow Visualization!



As $T\rightarrow 0$

There is much interest in quantum turbulence in ^{4}He and ^{3}He-B at temperatures where the density of normal fluid is negligible.

The search for appropriate experimental techniques for this temperature range poses major problems.

- Ion trapping can in principle measure line densities, but there are probably major problems; capture cross-sections are just being measured.
- Bubble states formed from triplet state He$_2$ molecules may prove powerful—but there are problems here too.
- Miniature temperature and pressure sensors are being developed.

T→ 0 continued

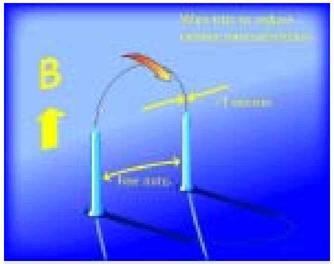
- ❑ Calorimetry: At very low temperatures the thermal energy in a superfluid can be very small, especially in ^{4}He. This means that turbulent energies can be comparable with the thermal energy.

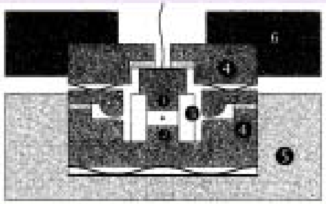
  Two consequencies:

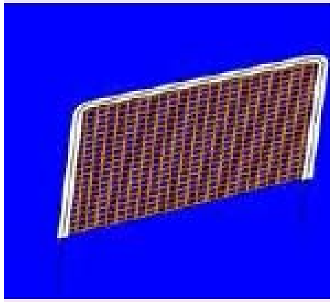
  - Decay of turbulence can be monitored by observing rise in temperature (good).
  - Continuous maintenance of steady-state turbulence is impossible (bad, because gain in sensitivity in a transducer from time-averaging is ruled out).

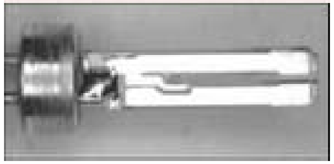
- ❑ Andreev reflection of thermal quasi-particles in ^{3}He-B by turbulent velocity fields: Quantitative measurements of vortex densities and the spatial distribution of vortices in ^{3}He-B possible at very low temperatures.

Producing Turbulence with Oscillating Bodies





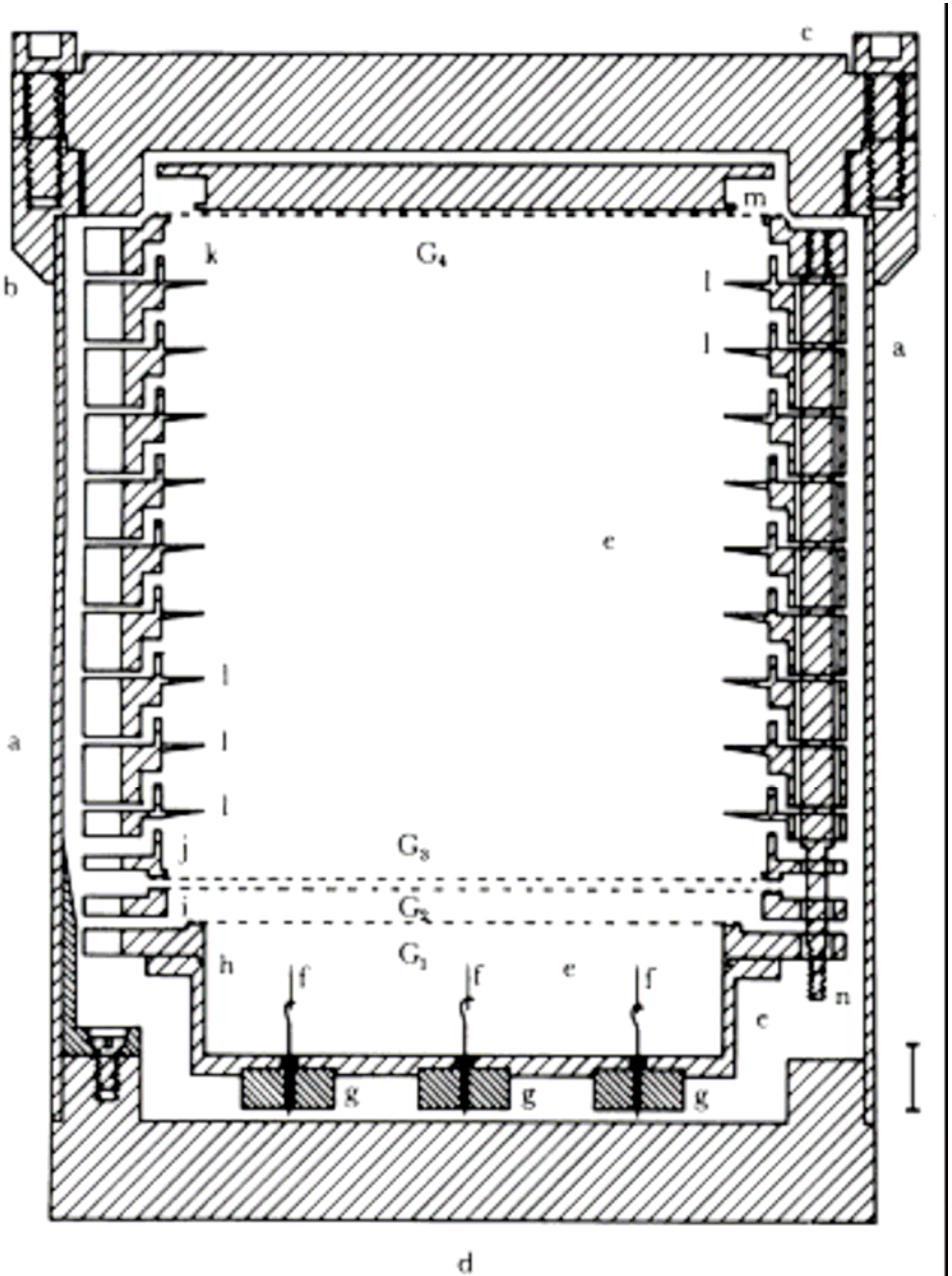




QT created by oscillation of –

- Wires: Grenoble, Helsinki, Kosiče, Lancaster, Moscow, Osaka, Sendai...
- Schoepe's sphere: Regensburg
- Grids: Lancaster ("square tennis racquet" as well as triple-capacitor)
- Quartz forks: Helsinki, Kosiče, Lancaster

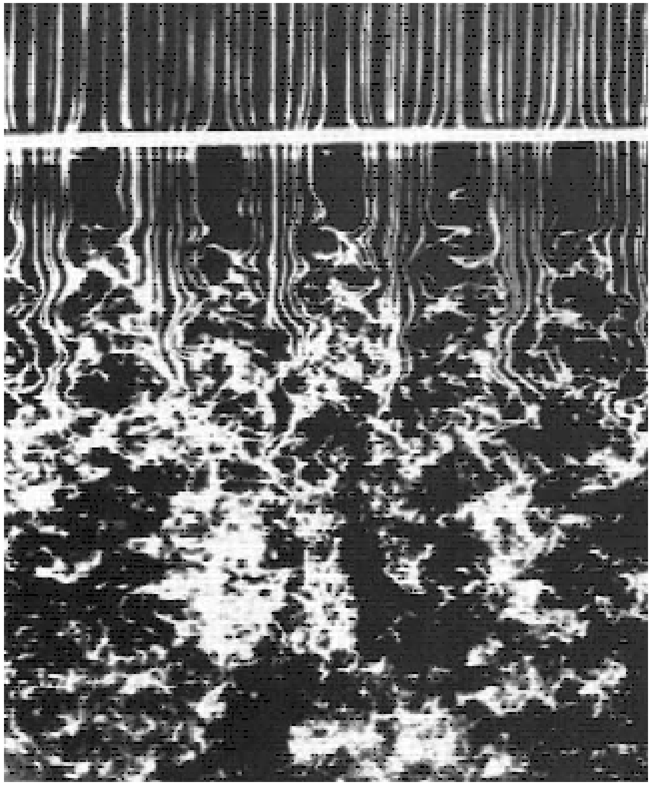
Lancaster “Big” Ion Cell

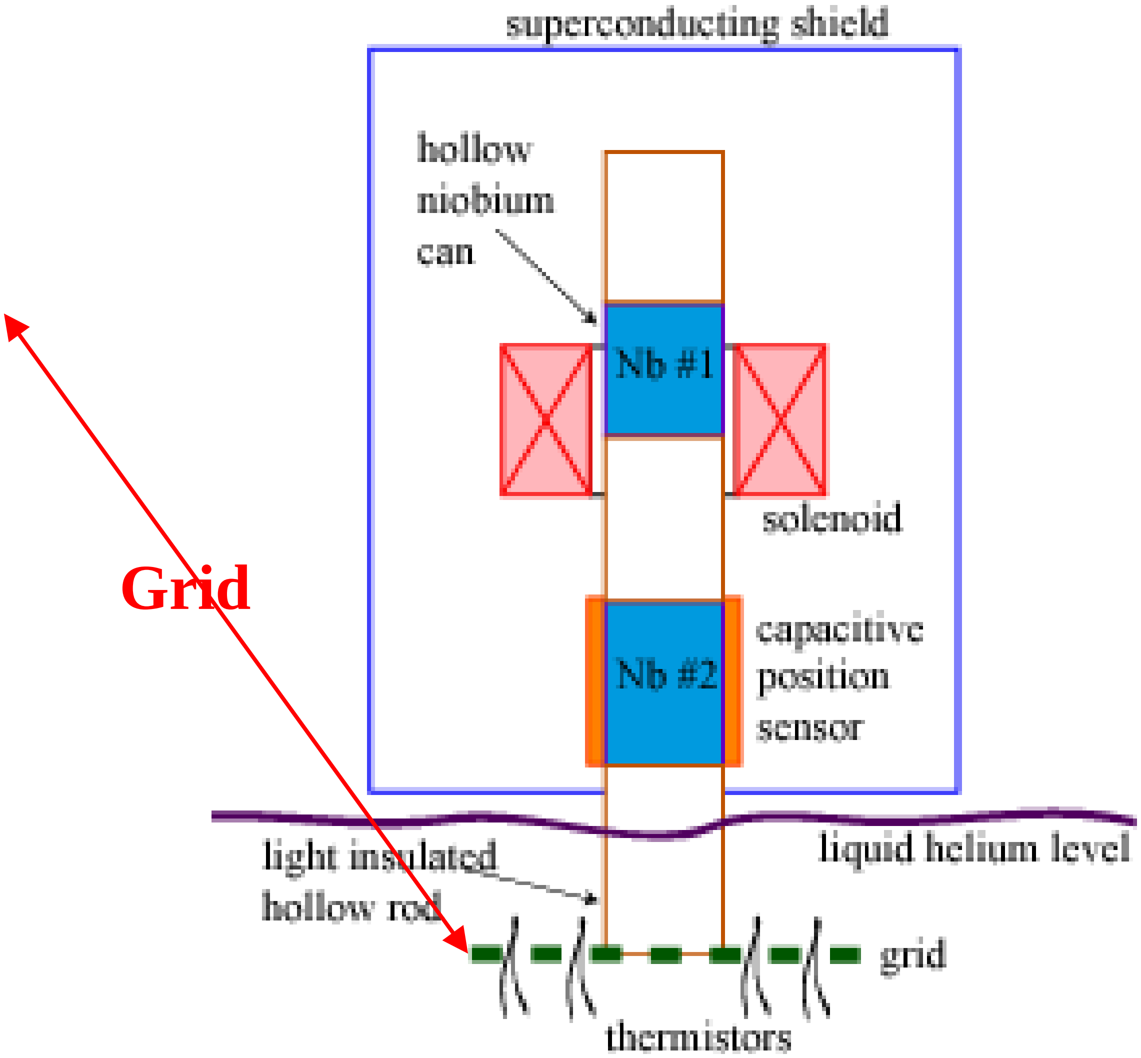


We want to study turbulence which has been well characterized classically and comparable to theory and simulations

Homogeneous Isotropic Turbulence

Pull grid at constant velocity





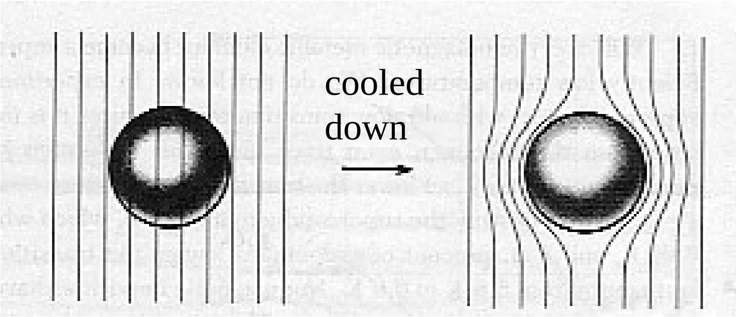
Meissner effect

Exhibiting diamagnetic properties to the *total* exclusion of all magnetic fields.

Superconducting sphere under constant magnetic field:

T > Tc

T < Tc

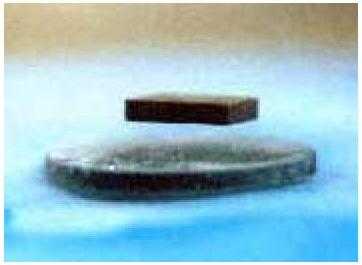


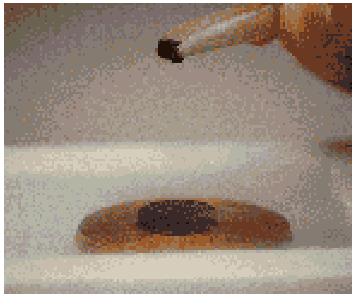
cooled down

-- the magnetic field lines are ejected from the sphere.

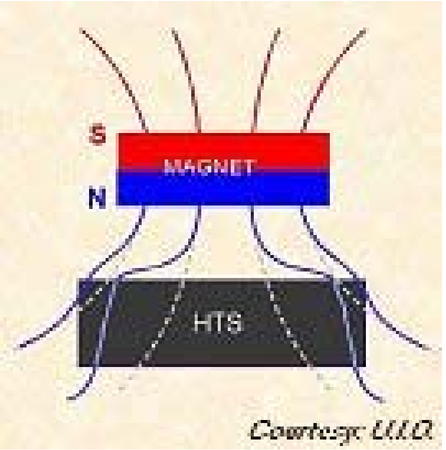
[Kittel 1996]

- **a magnet being levitated**



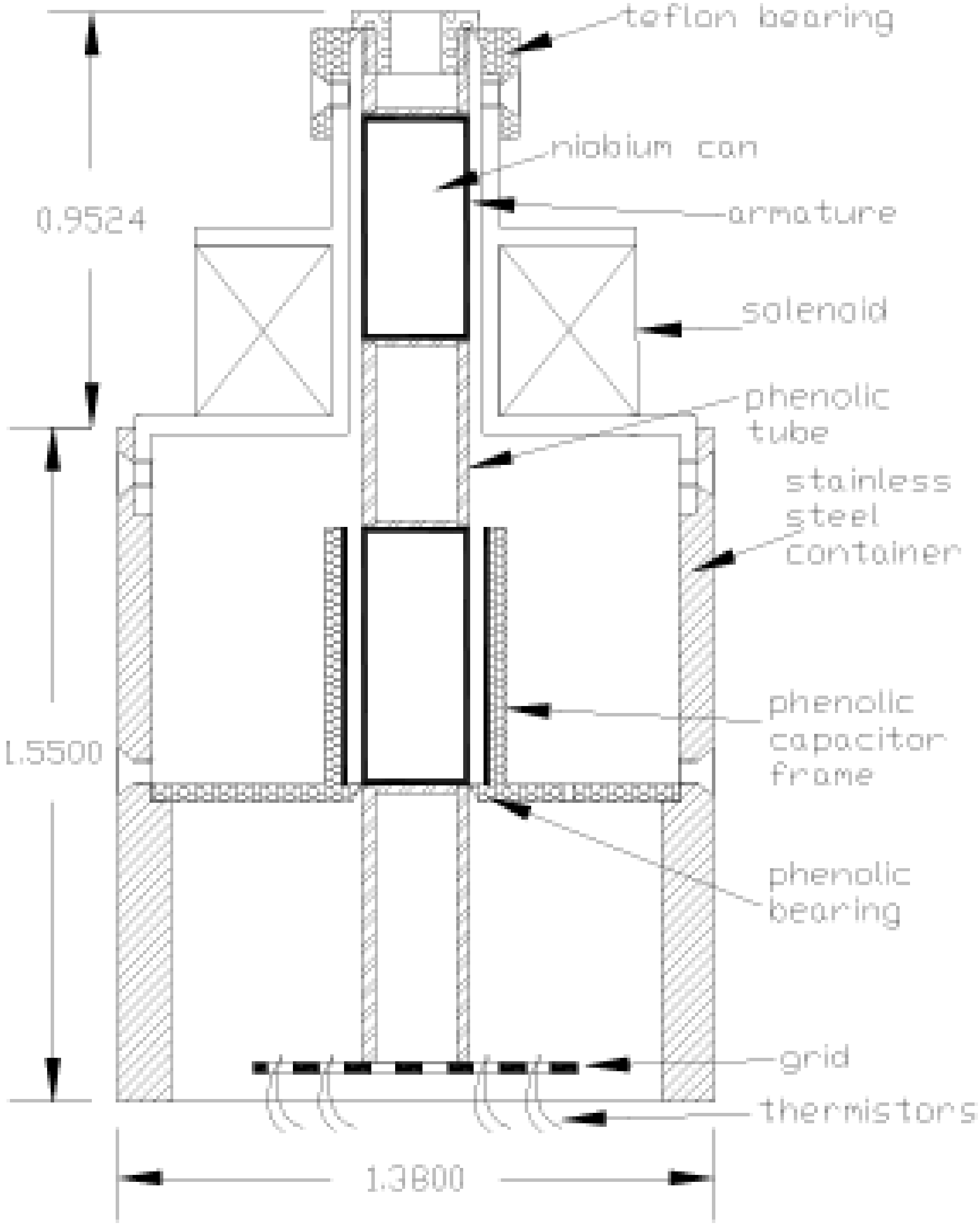


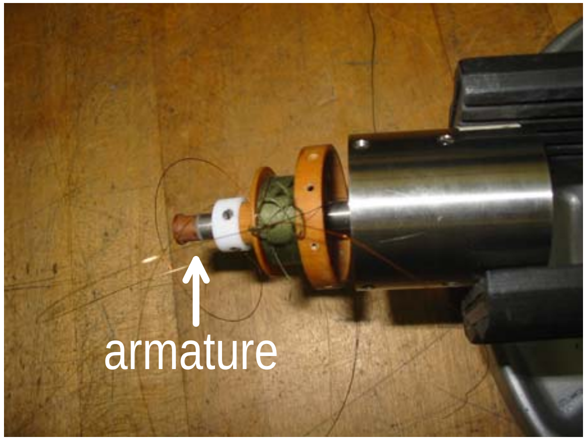
- **a magnet's flux lines folding around a superconductor.**

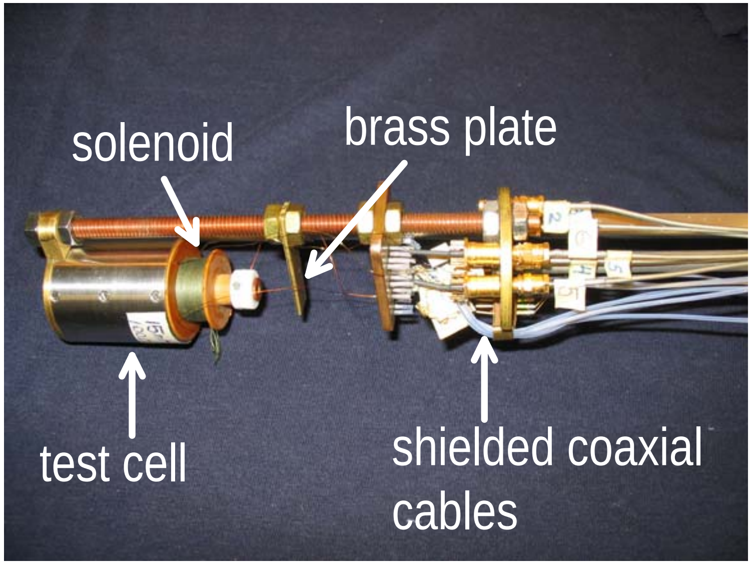


Courtesy: UUQ

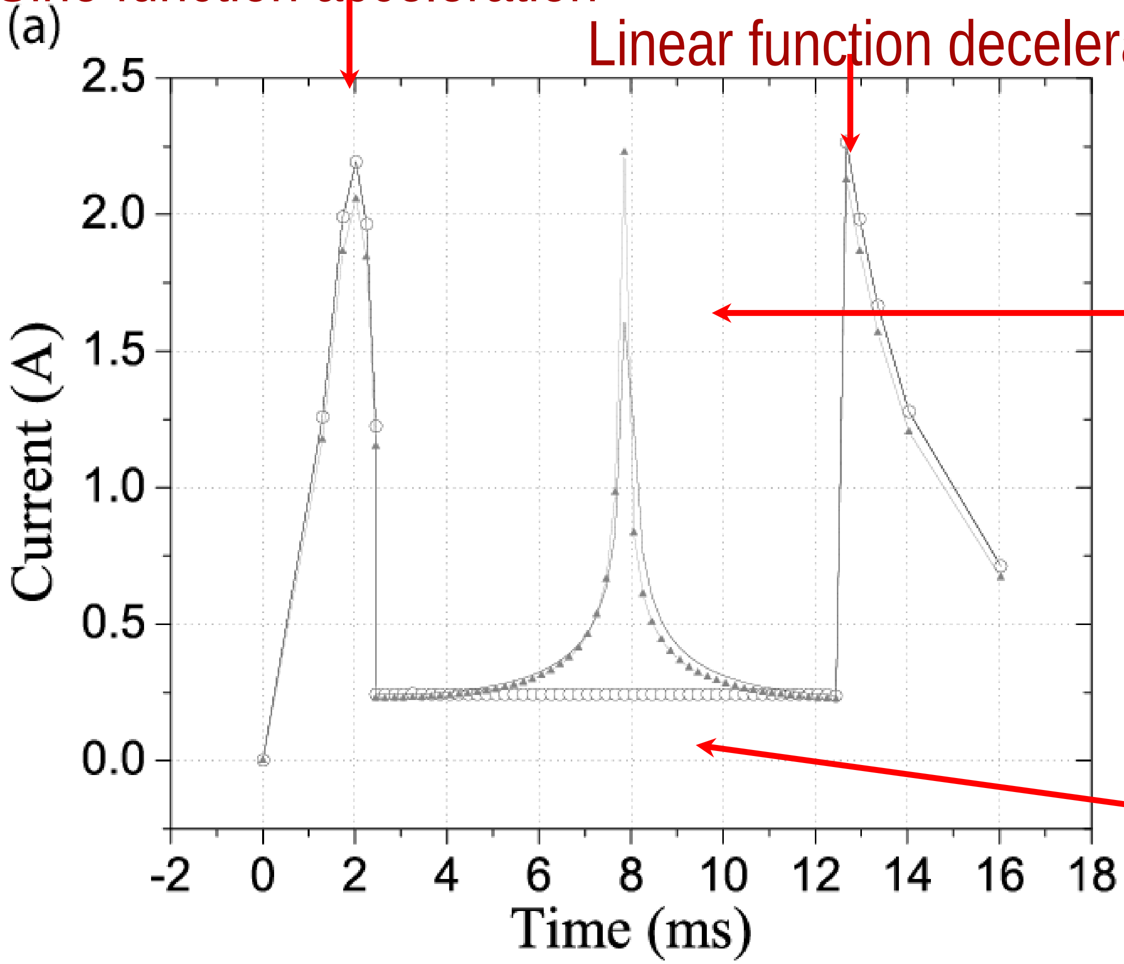
First realization of Motor







Current vs. Time Curves Superconducting Motor Simulation



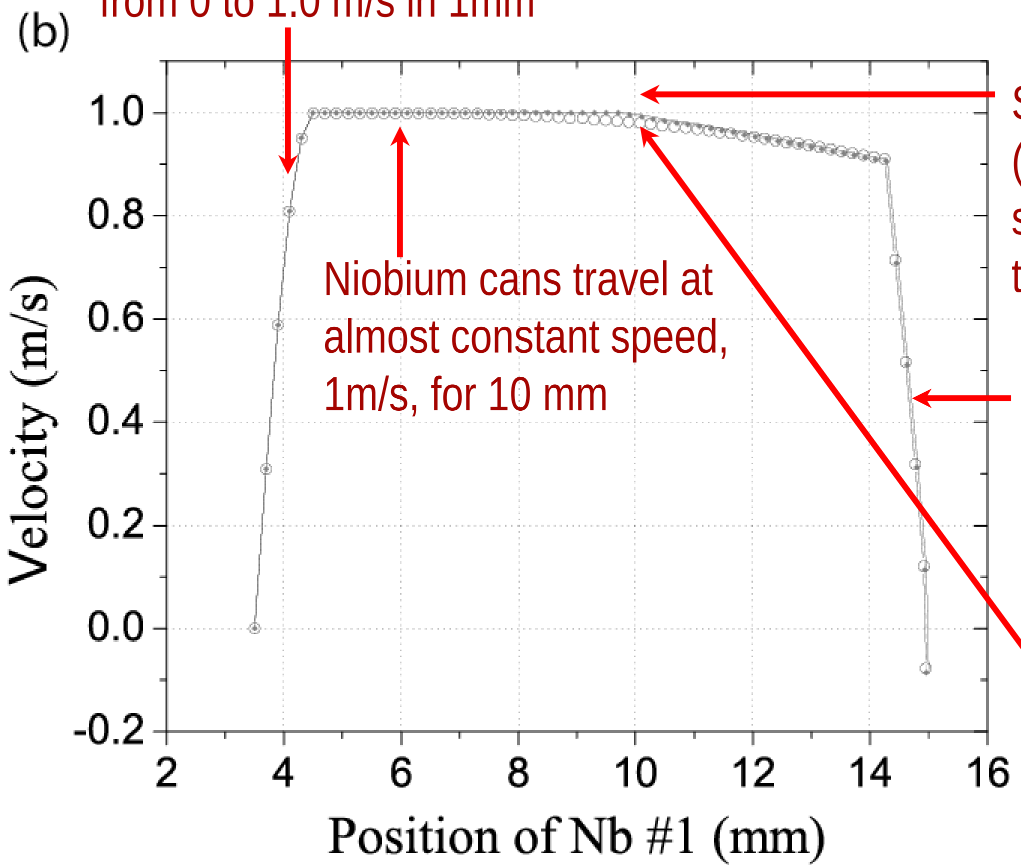
Sine function acceleration

Linear function deceleration

Peak due to the almost balanced magnetic forces on the two niobium cans at 8 ms, where each is almost equidistant from the ends of the solenoid

Inertia carries armature through this point without the central peak

Velocity vs. Position of Niobium Can #1 of Motor Simulation



Sine function acceleration from 0 to 1.0 m/s in 1mm

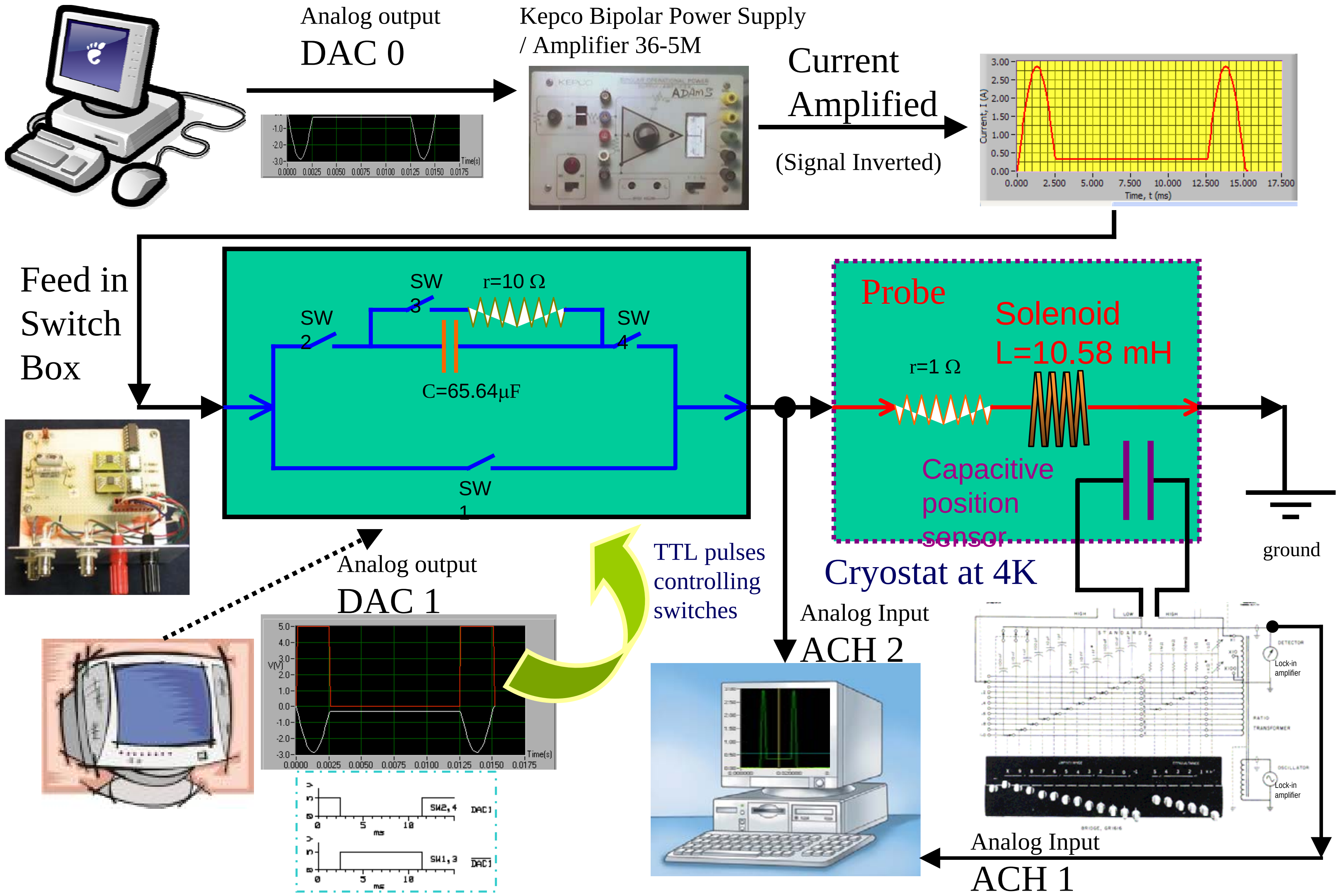
Niobium cans travel at almost constant speed, 1m/s, for 10 mm

Start to slow down at 10mm (Nb#2 closer to the solenoid, stronger magnetic force in the opposite direction)

Applying the third pulse produces the desired deceleration to rapidly stop the grid within 1mm

Motion without the central peak

Motor Electronics



Analog output
DAC 0

Kepco Bipolar Power Supply
/ Amplifier 36-5M

Current
Amplified

(Signal Inverted)

Feed in
Switch
Box

SW 3
r=10 Ω
SW 2
SW 4
C=65.64μF
SW 1

Probe
Solenoid
L=10.58 mH
r=1 Ω
Capacitive
position
sensor

ground

Cryostat at 4K

Analog output
DAC 1

TTL pulses
controlling
switches

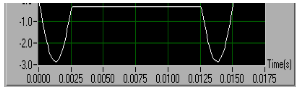
Analog Input
ACH 2

Analog Input
ACH 1

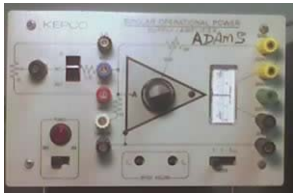
Electronics-No Switch Box

Analog output

DAC 0

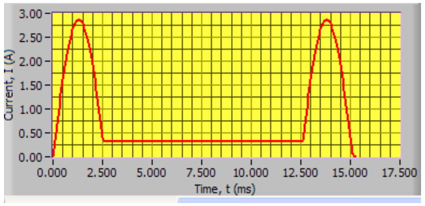


Kepco Bipolar Power Supply / Amplifier 36-5M

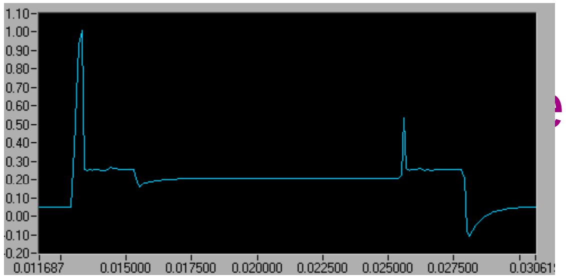


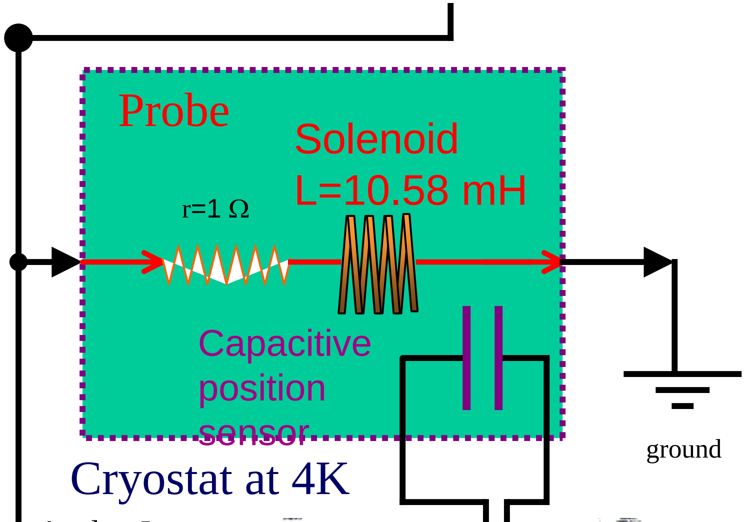
Current Amplified

(Signal Inverted)



We can also use without the switch box because of our robust current amplifier filtering out the possibly produced spikes due to the fast





Probe

Solenoid

L=10.58 mH

r=1 Ω

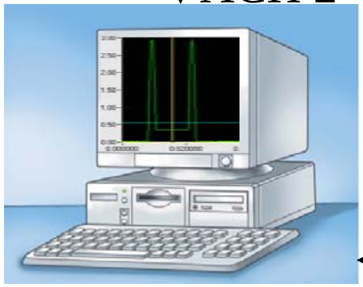
Capacitive position sensor

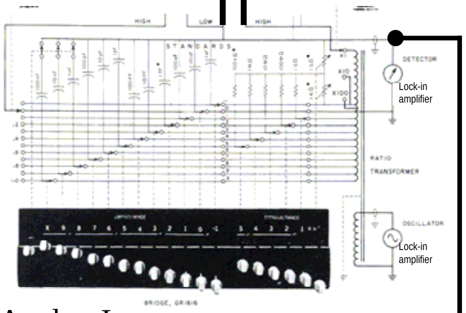
Cryostat at 4K

ground

Analog Input

ACH 2

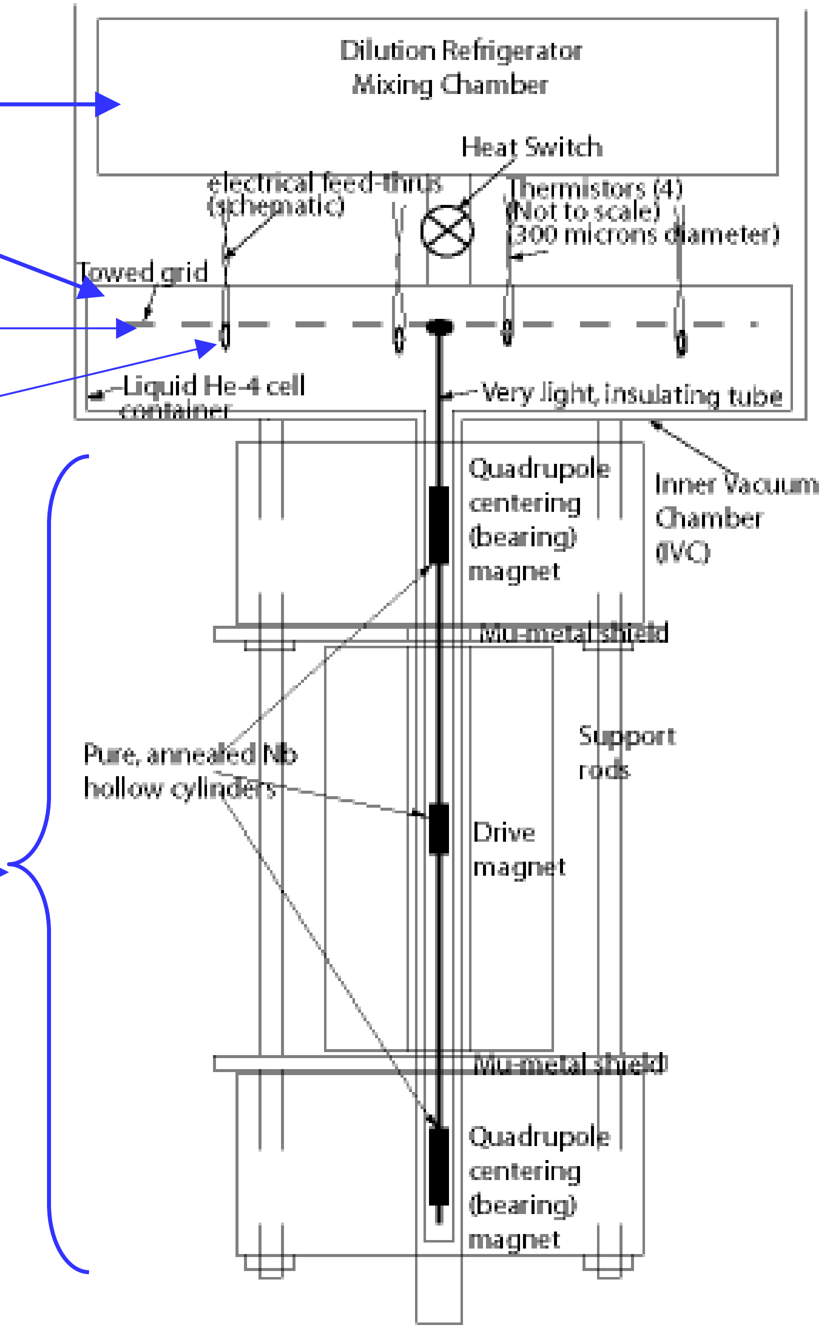




Analog Input

ACH 1

A more sophisticated Motor



Mixing chamber of refrigerator

Liquid helium

Grid

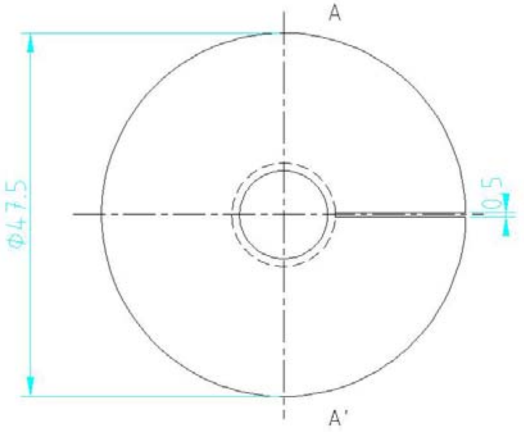
Thermistor

Motor

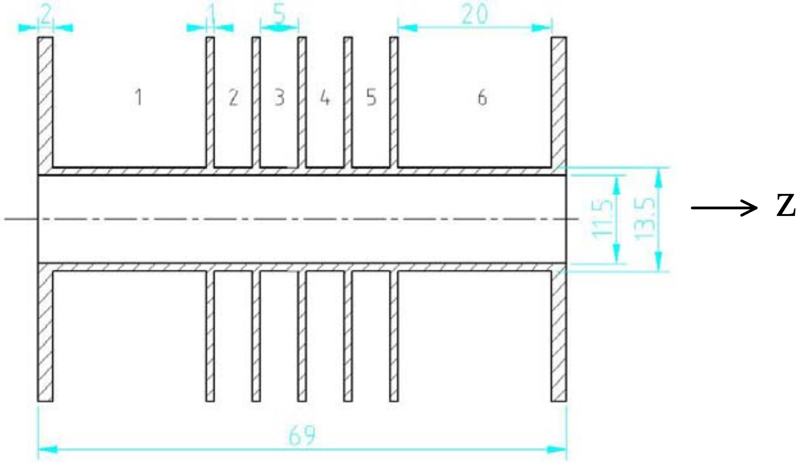
D. Charalambous,
P.C. Hendry,
P.V.E. McClintock,
W.F. Vinen,
M. Giltrow

Drive Magnet

Top View

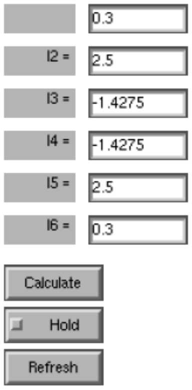


Side View (six coaxial coils)



The central superconducting Nb cylinder (tube and gird) is levitated and shifted by the magnetic field minimum along the coil axis (z).

Changing the magnitudes and directions of the currents of the six coils moves the position of the magnetic field minimum along the z axis.



0.3

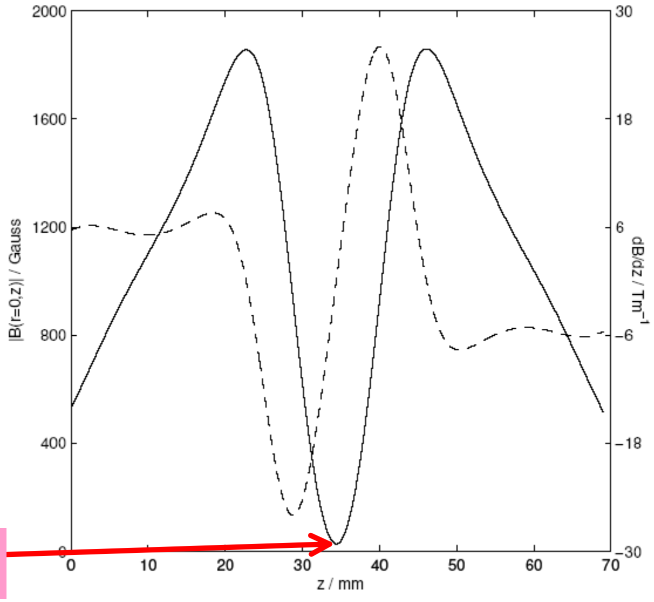
I2 = 2.5

I3 = -1.4275

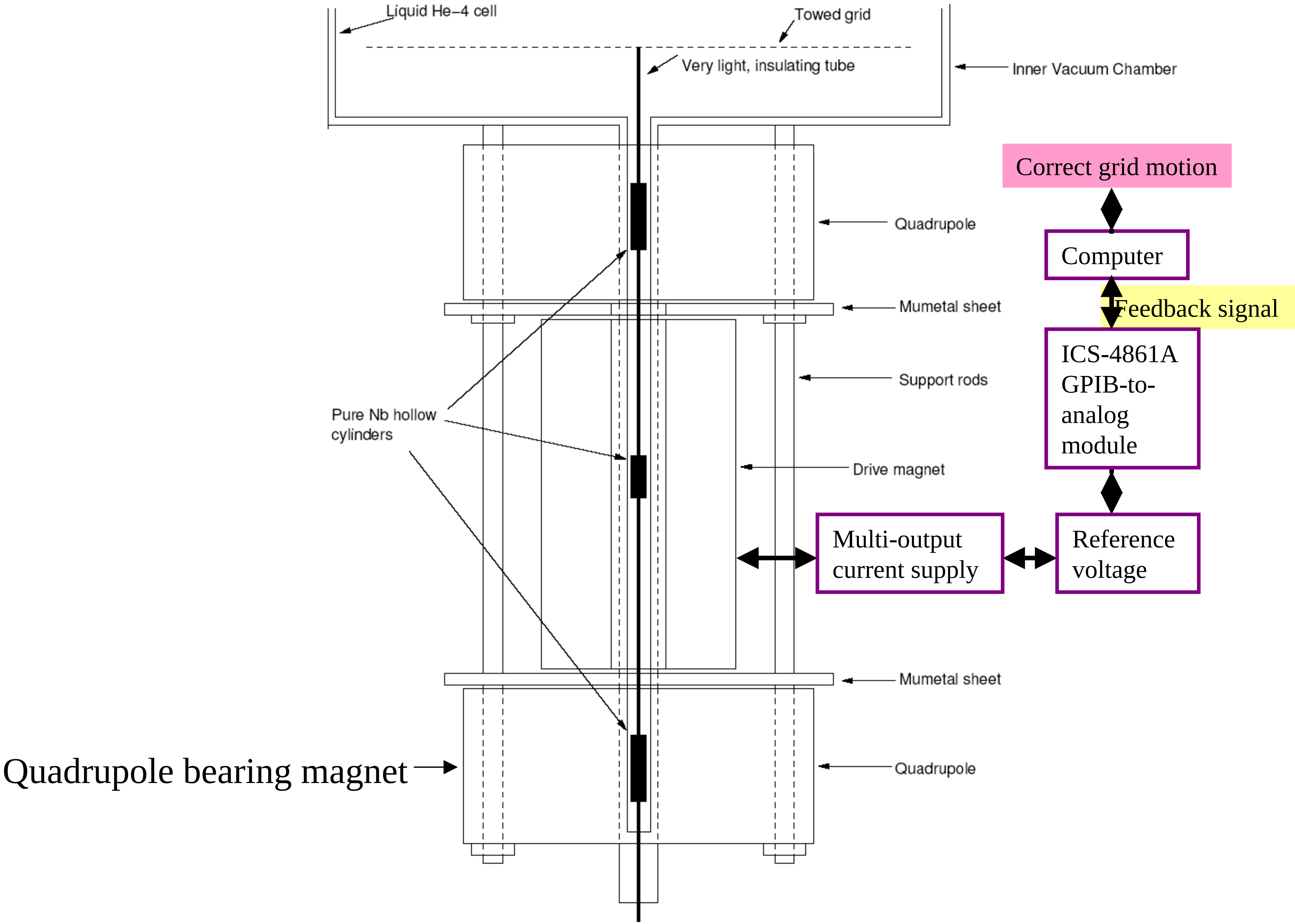
I4 = -1.4275

I5 = 2.5

I6 = 0.3



Sharp magnetic field minimum



Correct grid motion

Computer

Feedback signal

ICS-4861A GPIB-to-analog module

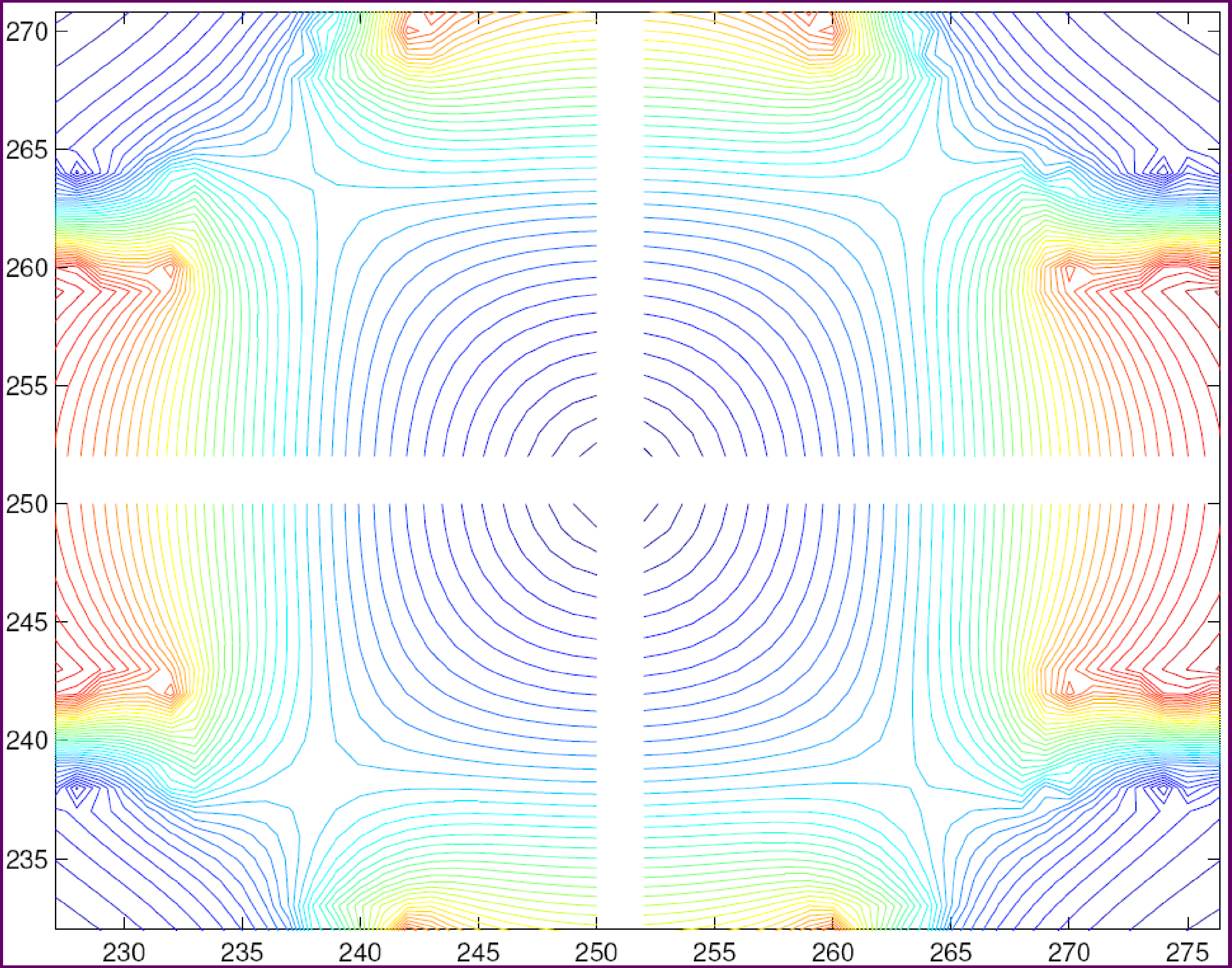
Multi-output current supply

Reference voltage

Quadrupole bearing magnet

Quadrupole Magnet

-- provide a lateral (radial) force to keep the end-Nb-cylinders/central tube in position



Calorimetry Probe Development

Thermistor Characteristics

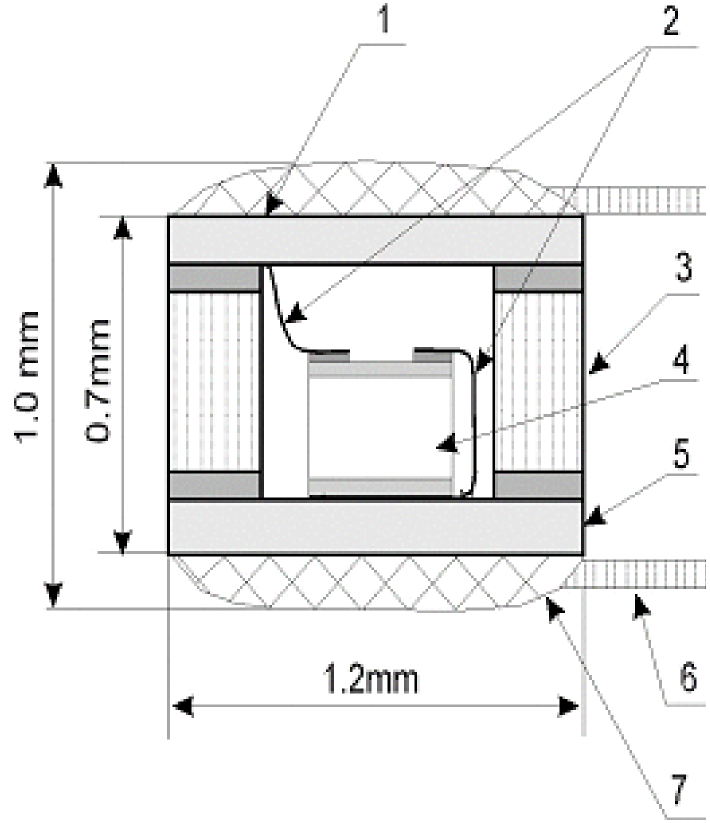
- Operating temperature: 10 – 100 mK
- Sensitivity: $\delta T \sim 10^{-4}$ mK
- Short response time: ~ 1 ms
- Small mass & good thermal contact.
- Ease of manufacture

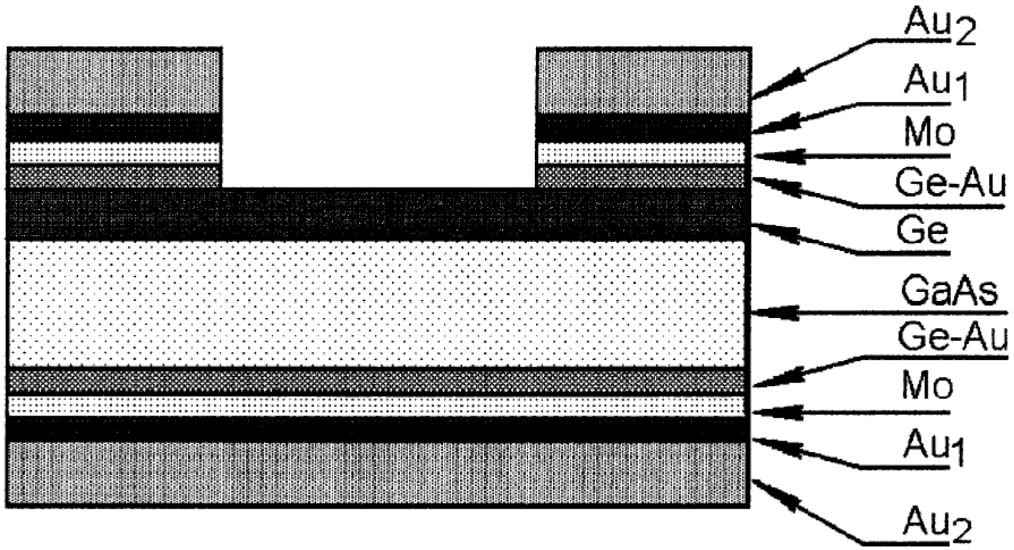
Use computer chip fabrication technique: V. Mitin

http://microsensor.com.ua/products.html

Sensor Package Construction

- Ge/GaAs thermistors **300 μm** square by **150 μm** thick.





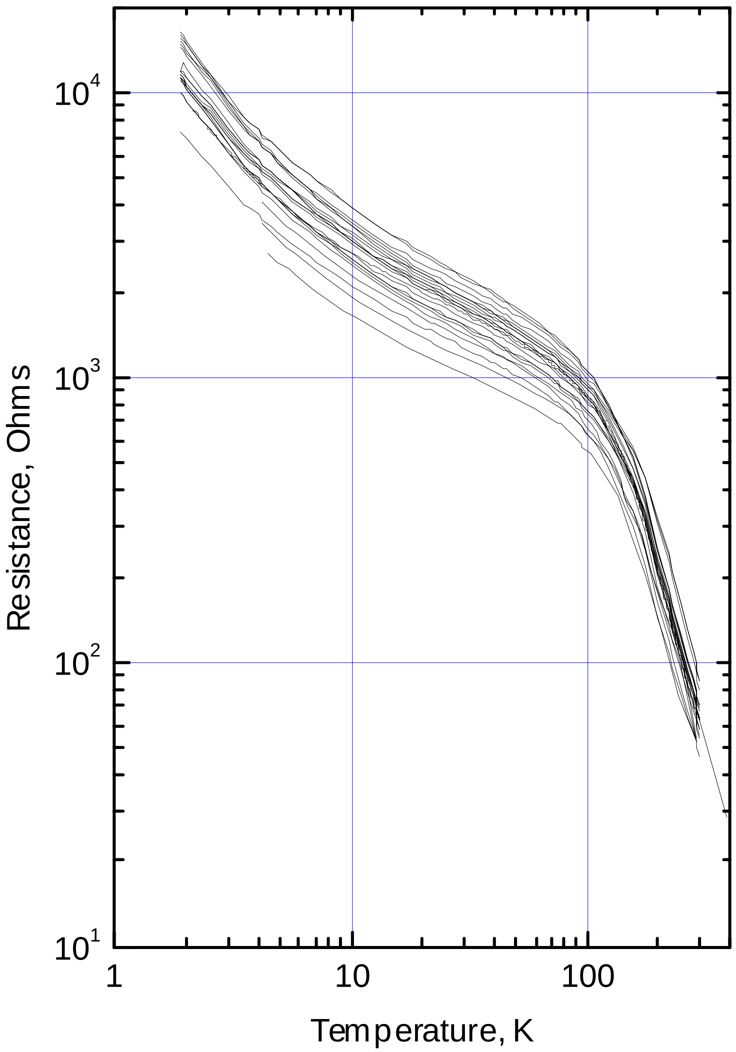
Ge/GaAs Thin Film Element

1 and 5: copper discs; 2: gold strip; 3: corundum cylinder;
4: Ge/GaAs sensitive element; 6: copper wire; 7- tin.

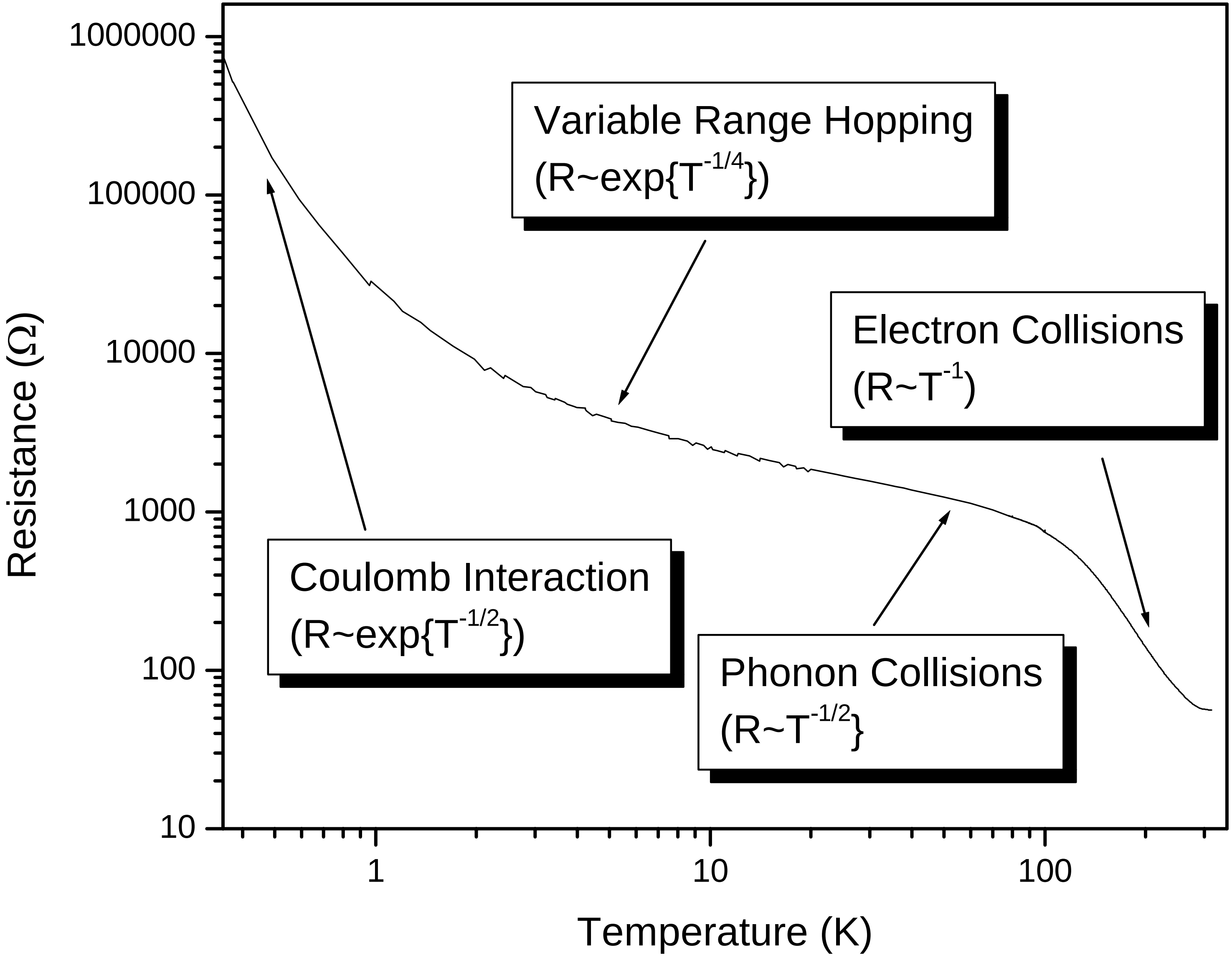
Advantages of Thin Film Technology

Mass Production/Consistency

- Each wafer will generate sensors with very similar properties
- Resistance measurements made on a single batch over the range 10K – 150K →
- A single fixed point measurement at 4.2K will approximate the sensors properties if the entire curve for any one sensor from the batch is known



Conduction in a doped semiconductor



Variable Range Hopping
($R \sim \exp\{T^{-1/4}\}$)

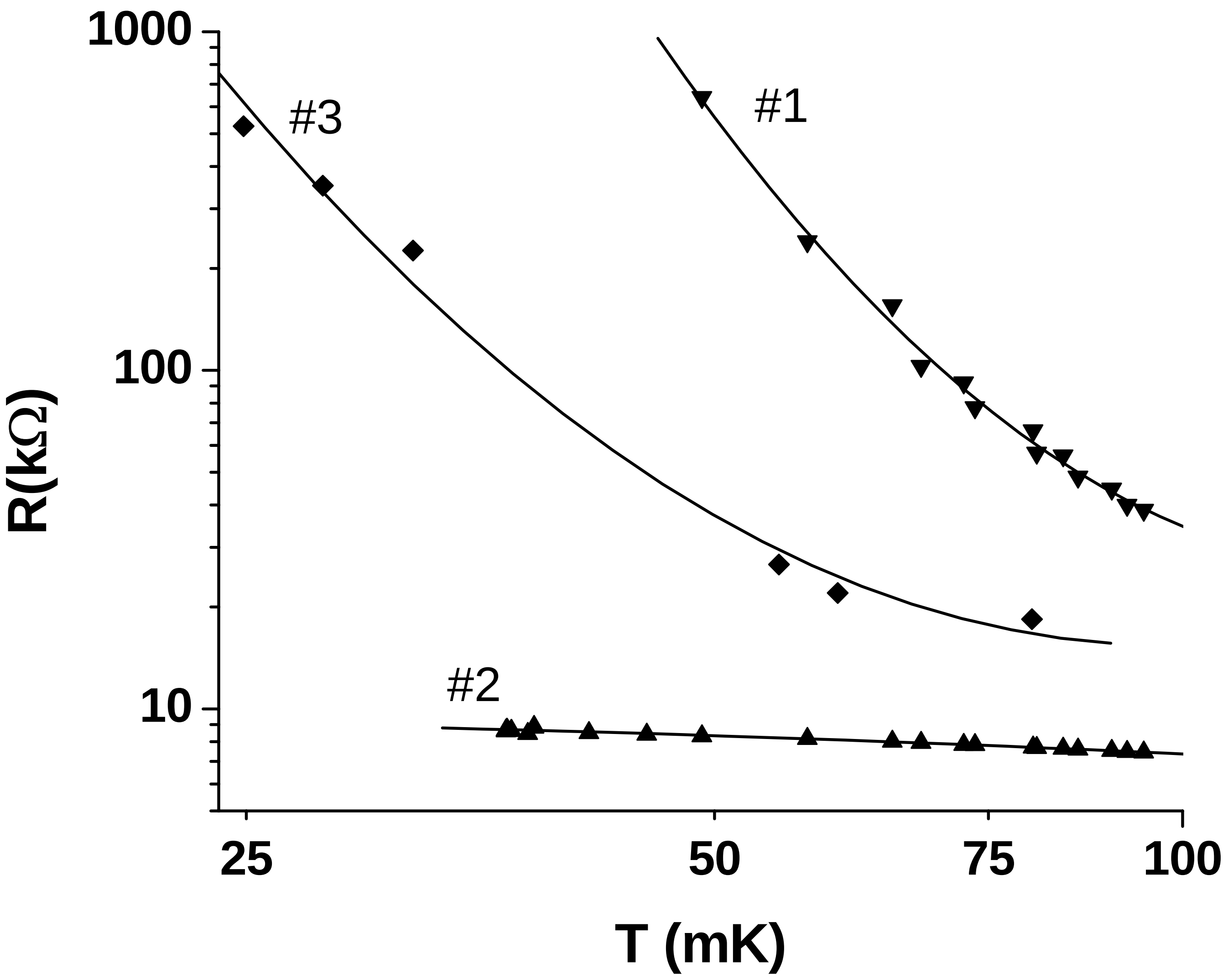
Electron Collisions
($R \sim T^{-1}$)

Coulomb Interaction
($R \sim \exp\{T^{-1/2}\}$)

Phonon Collisions
($R \sim T^{-1/2}\}$

Thermistor R vs. T Development Work

Tune characteristics by heat treatment



Another Probe or two or three

Pressure Transducer Requirements

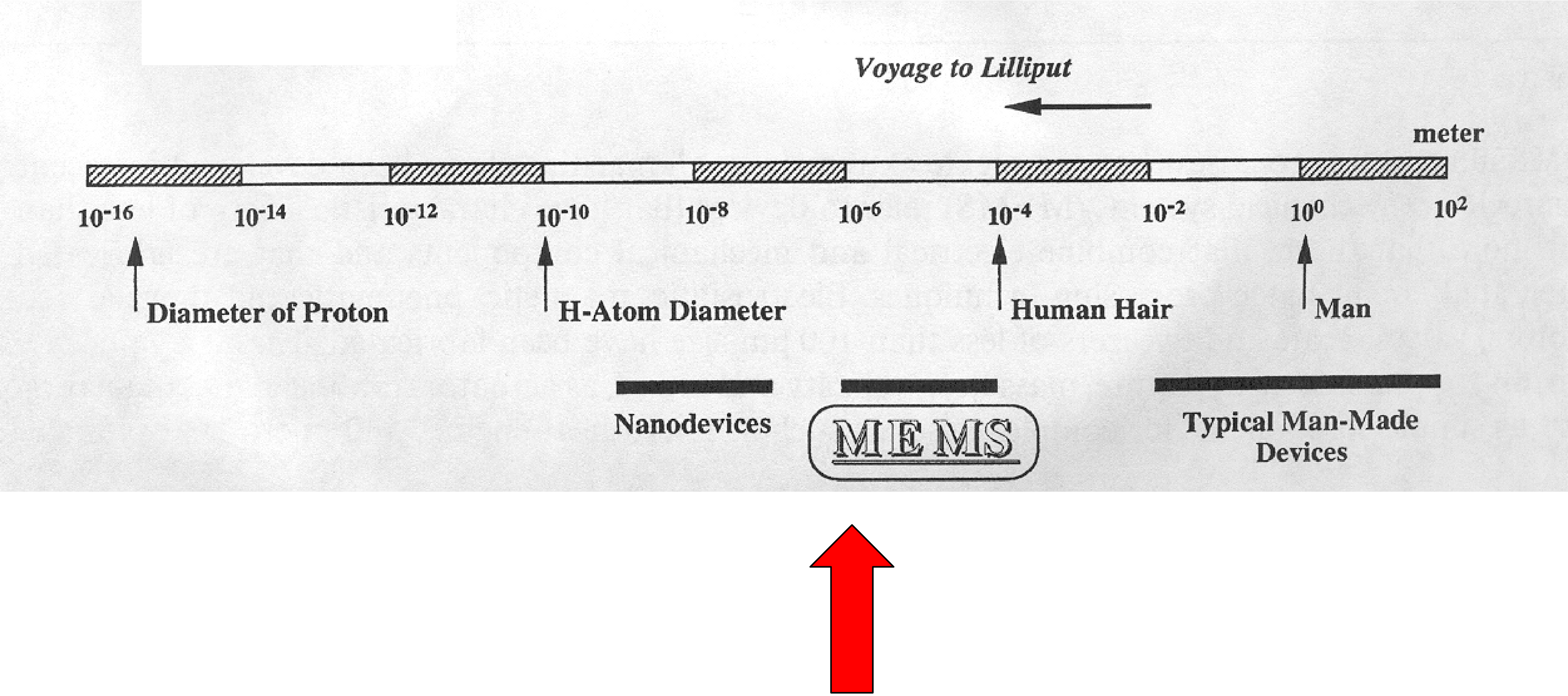
- sampling on micron scale
- sensitivity: 0.1 Pascal
- fast: 1 msec
- function at low temperatures (20 – 100 mK)
- transduction: as simple as possible

MEMS Technology Pressure Sensors

- Piezo-resistive
- Capacitive
- Optical

MEMS Technology

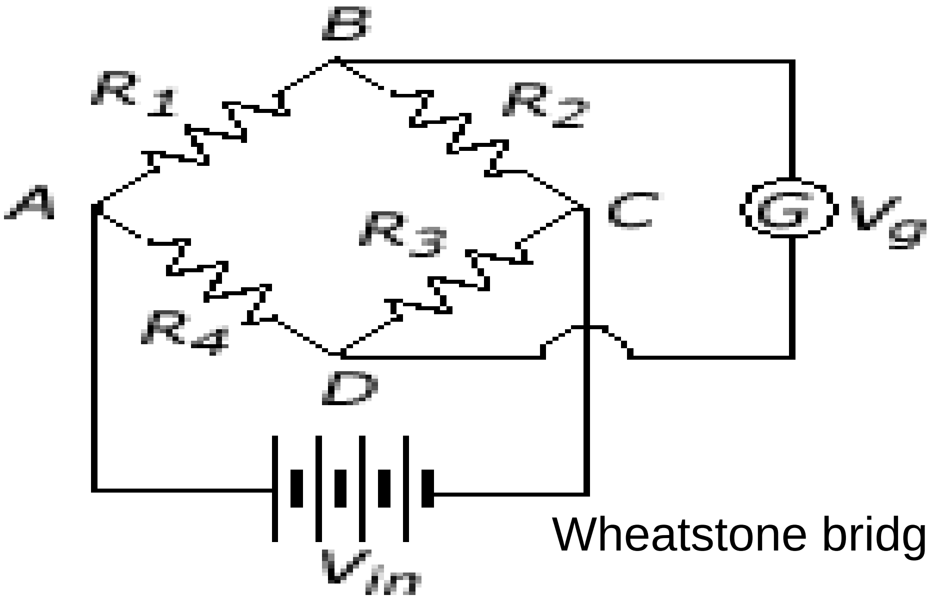
Perfect size range



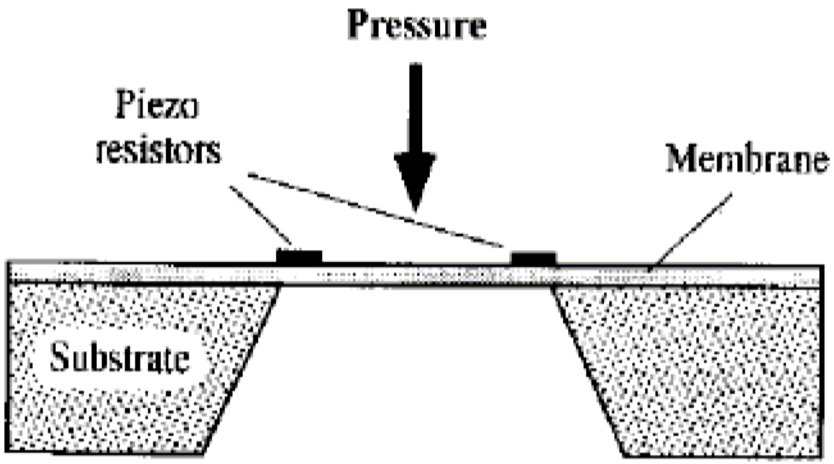
1 µm

Design Of Piezo-resistive Pressure Sensors

- Typical design: 4 piezo-resistors in Wheatstone bridge on a diaphragm
- diaphragm deflects from applied pressure causing the deformation of the piezo-resistors mounted on the surface



Wheatstone bridge



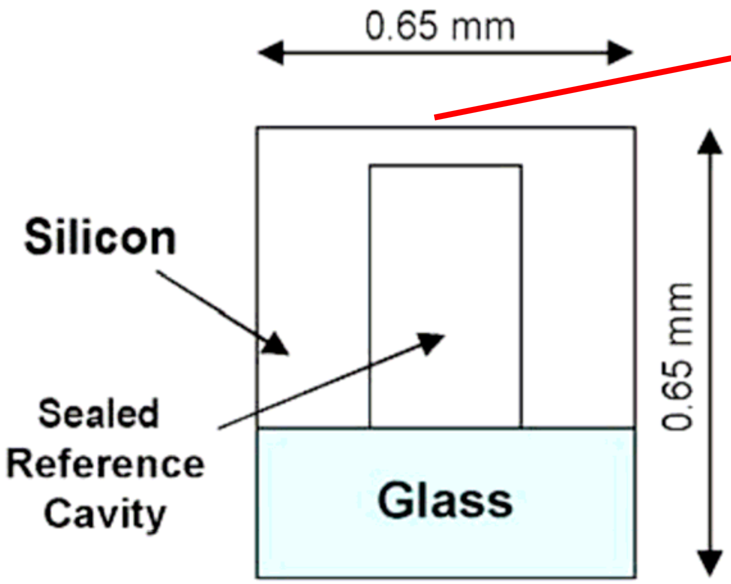
Piezo-resistive Pressure Sensor SM5108

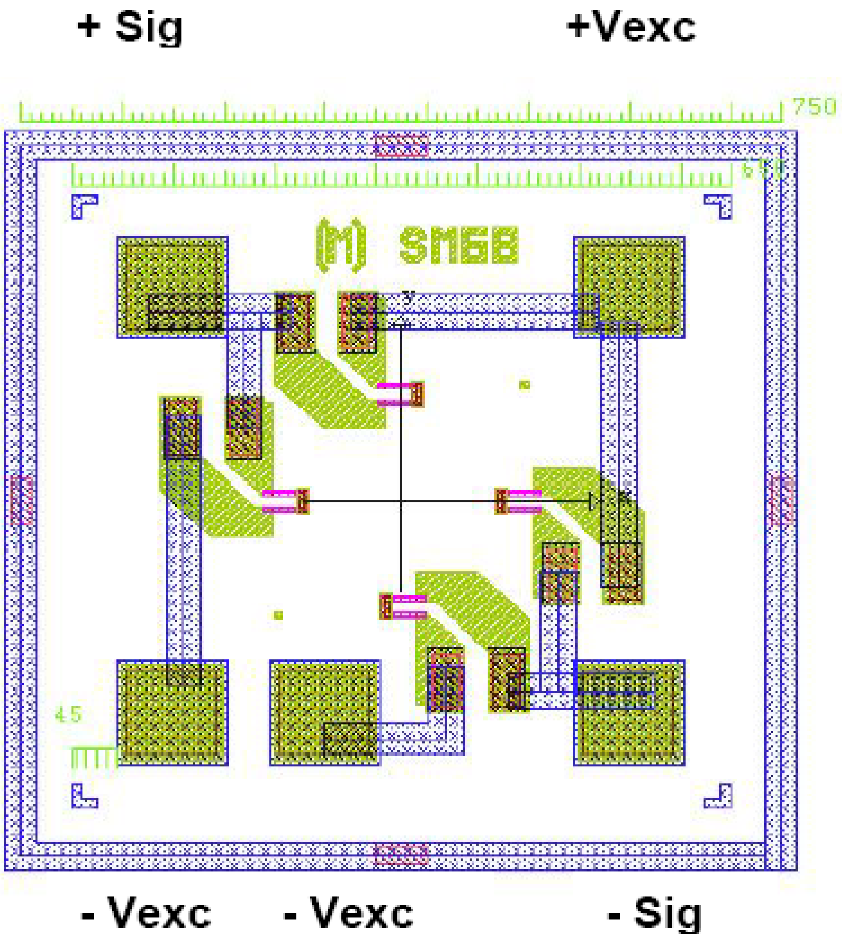
Semiconductor resistors joined by aluminum conductors in bridge configuration

Resistors placed on diaphragm

Two strained parallel to I

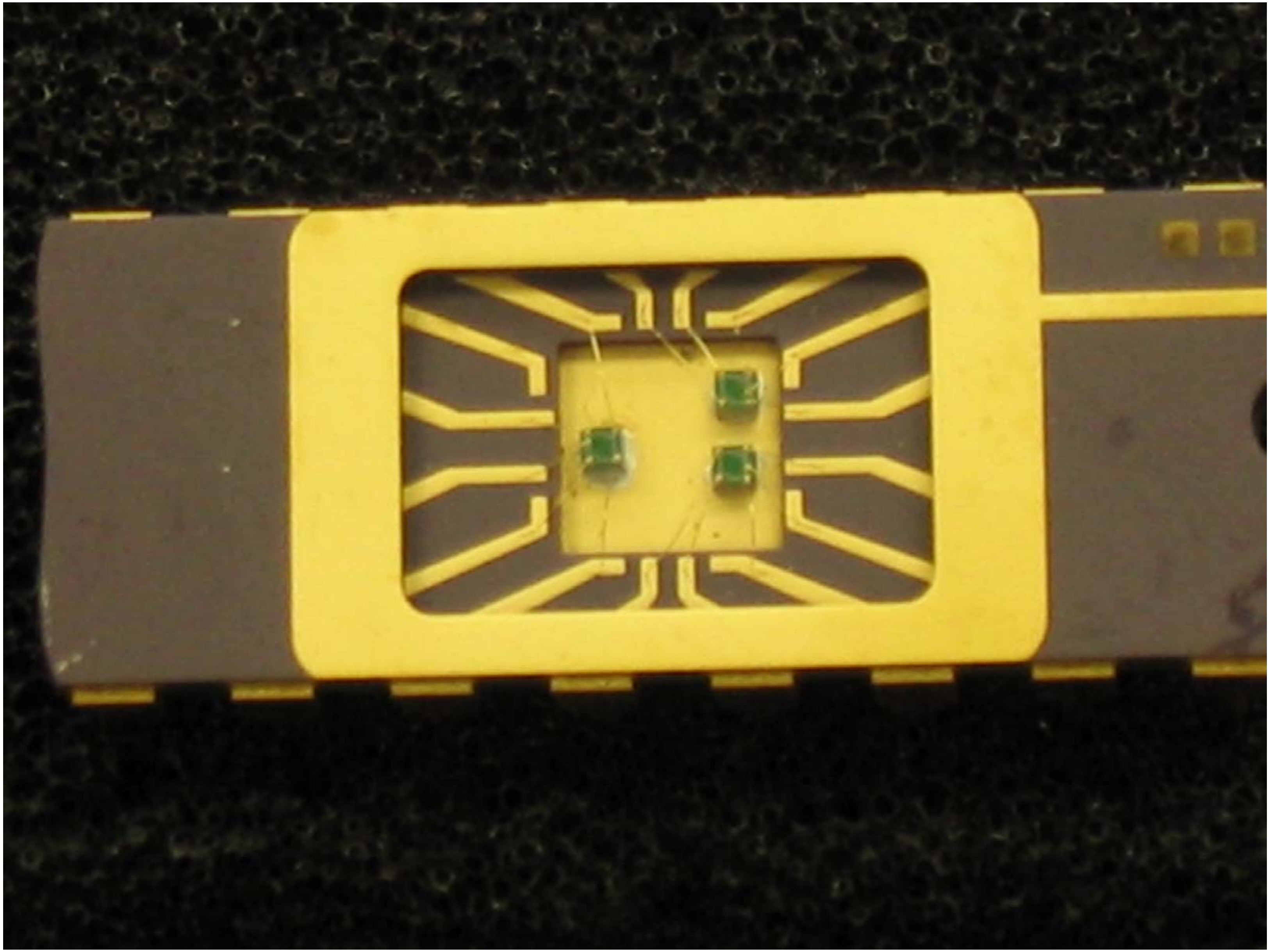
Two strained perpendicular to I





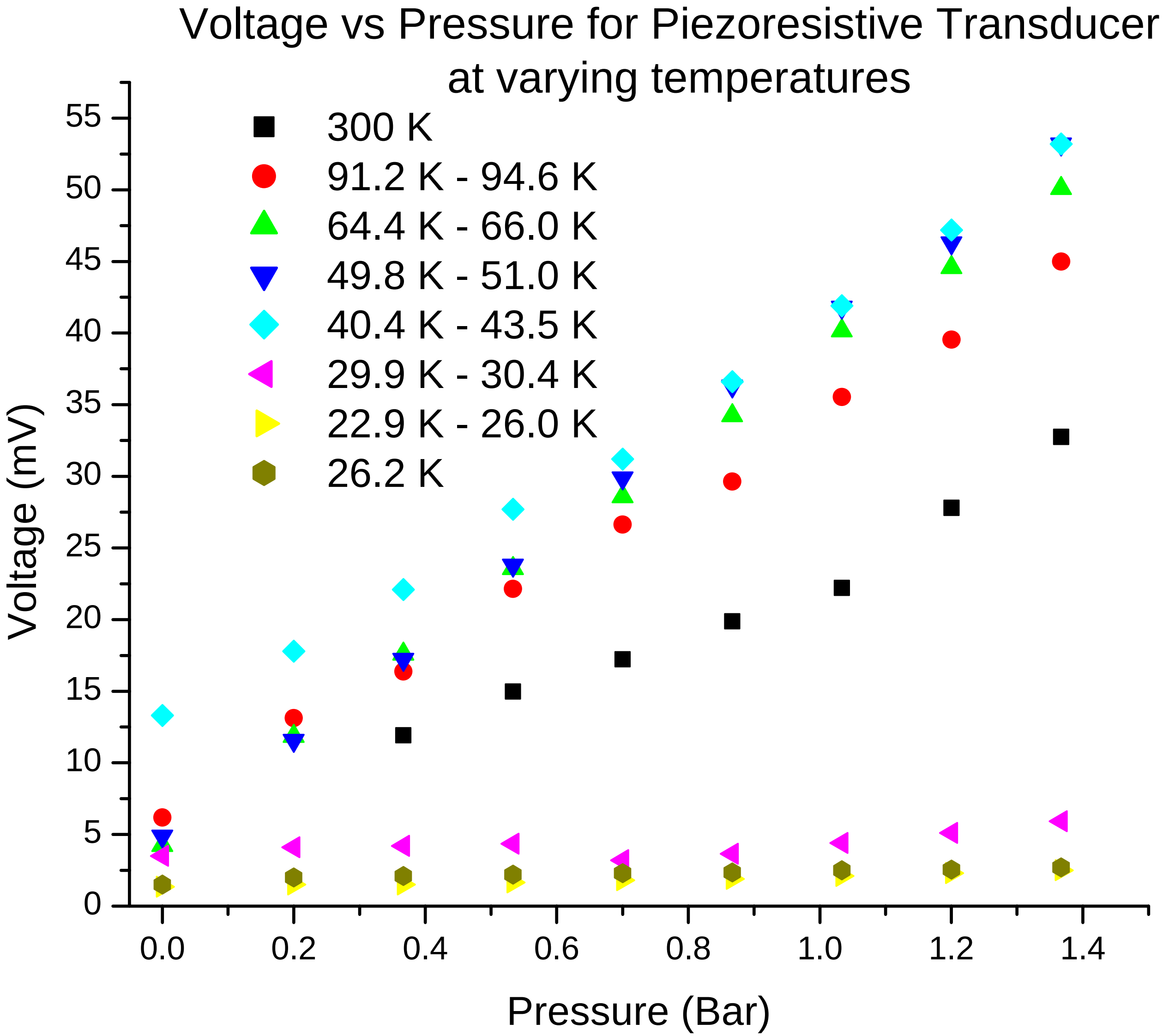
Manufactured by Silicon Microstructures, Inc.

Piezo-resistive Pressure Sensor SM5108

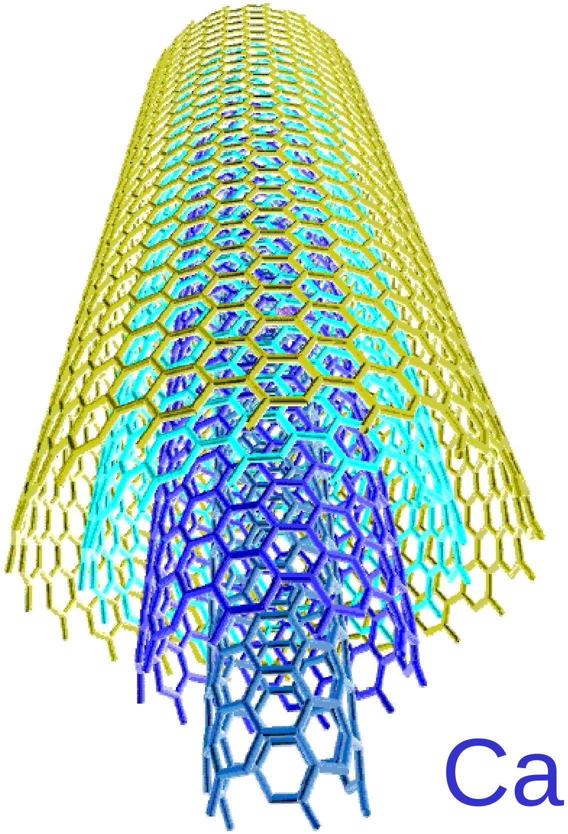


Drawbacks of Piezo-resistive Pressure Sensors-Results

- Relatively low sensitivity
- Large temperature dependence temperature compensation necessary



Voltage vs Pressure for Piezoresistive Transducer at varying temperatures



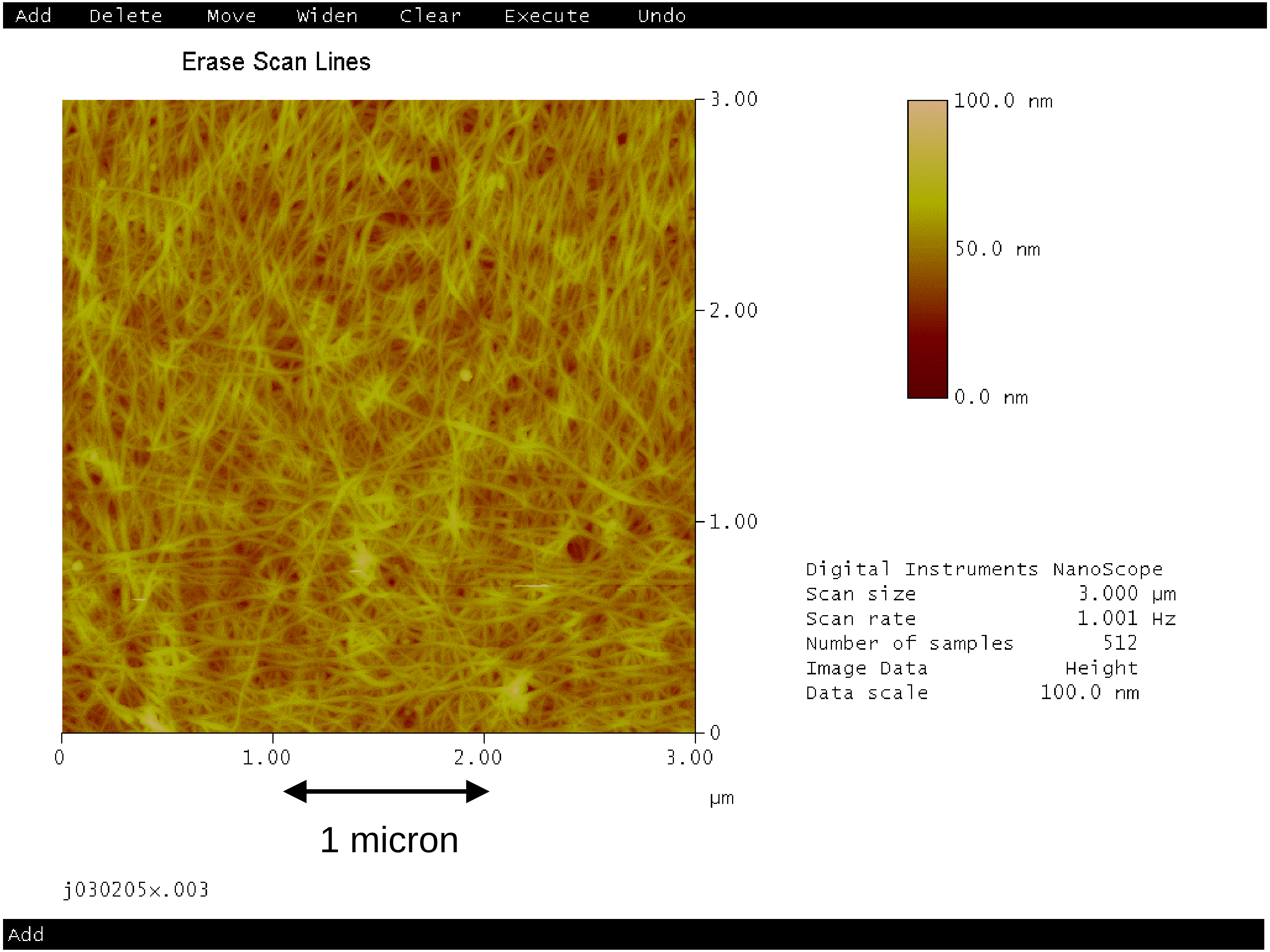
Nanotube/Film Technology

- Small
- Strong
- Conducting
- But not too conducting
- Elastic
- Stick to some surfaces

Can be used for

Thermometers
Heaters
Strain gauges
Capacitor plates
Flow meters
Turbulence detectors

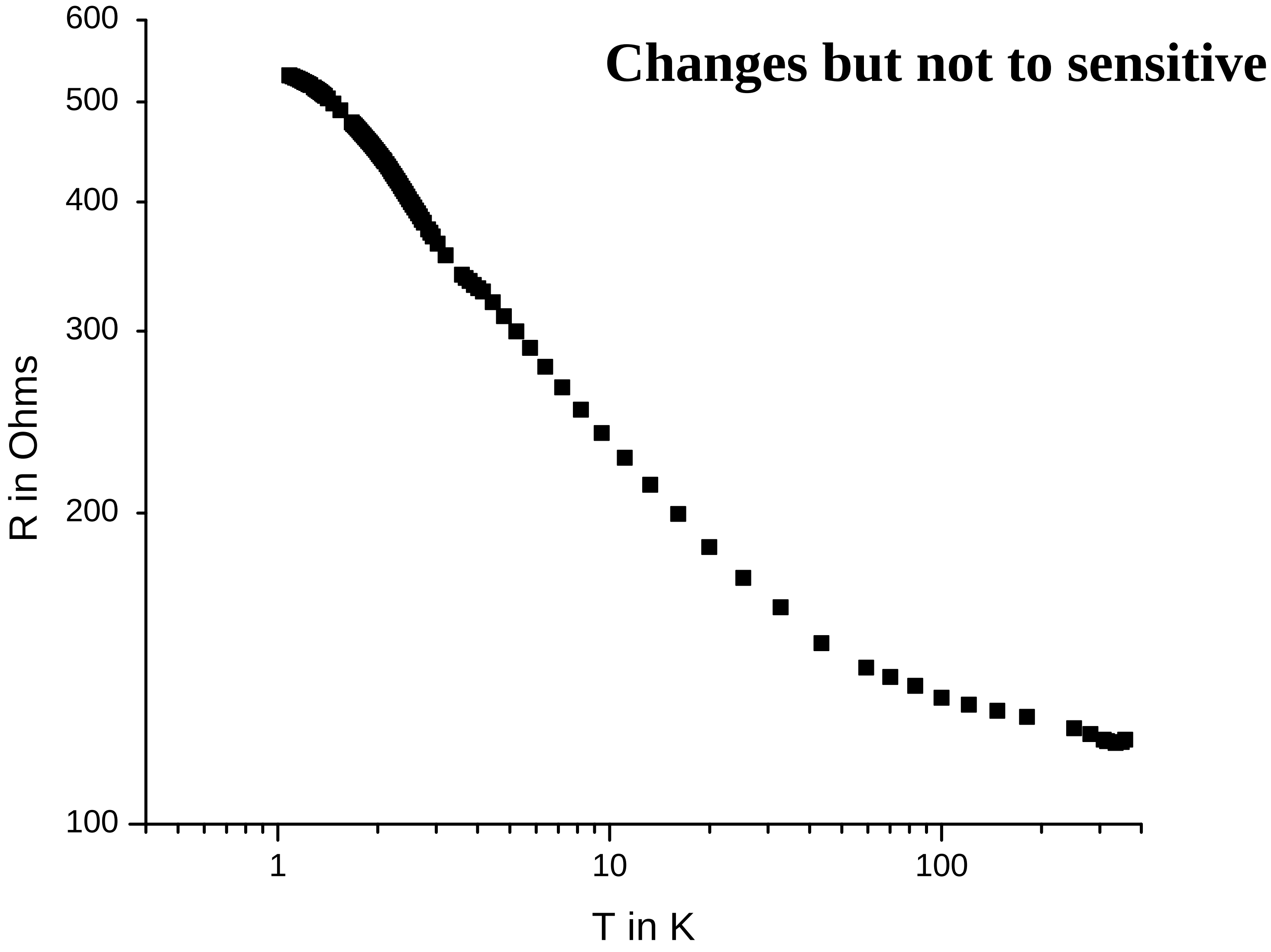
Nanotube film AFM Image



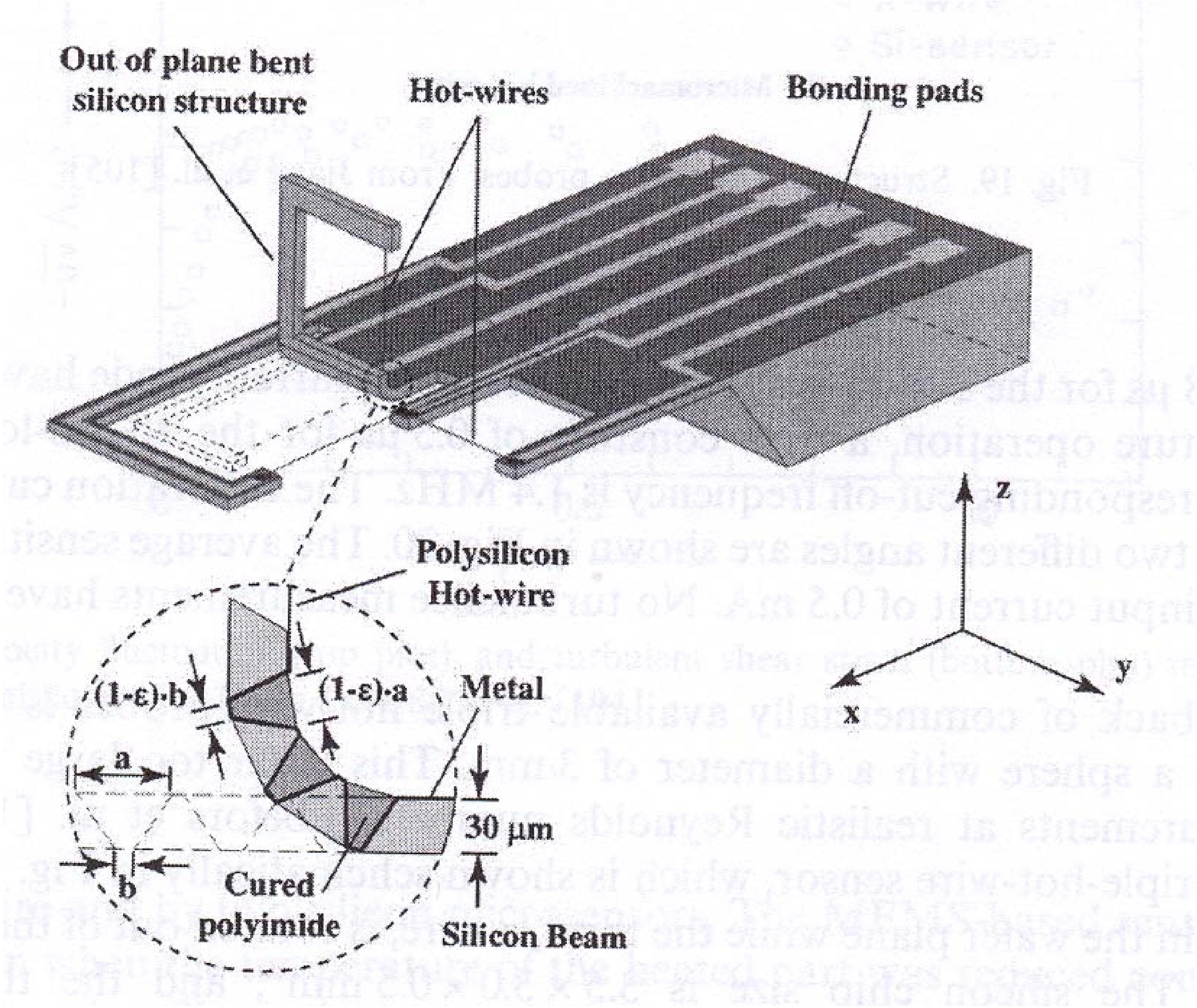
```
Digital Instruments NanoScope
Scan size              3.000 µm
Scan rate              1.001 Hz
Number of samples        512
Image Data             Height
Data scale            100.0 nm
```

1 micron

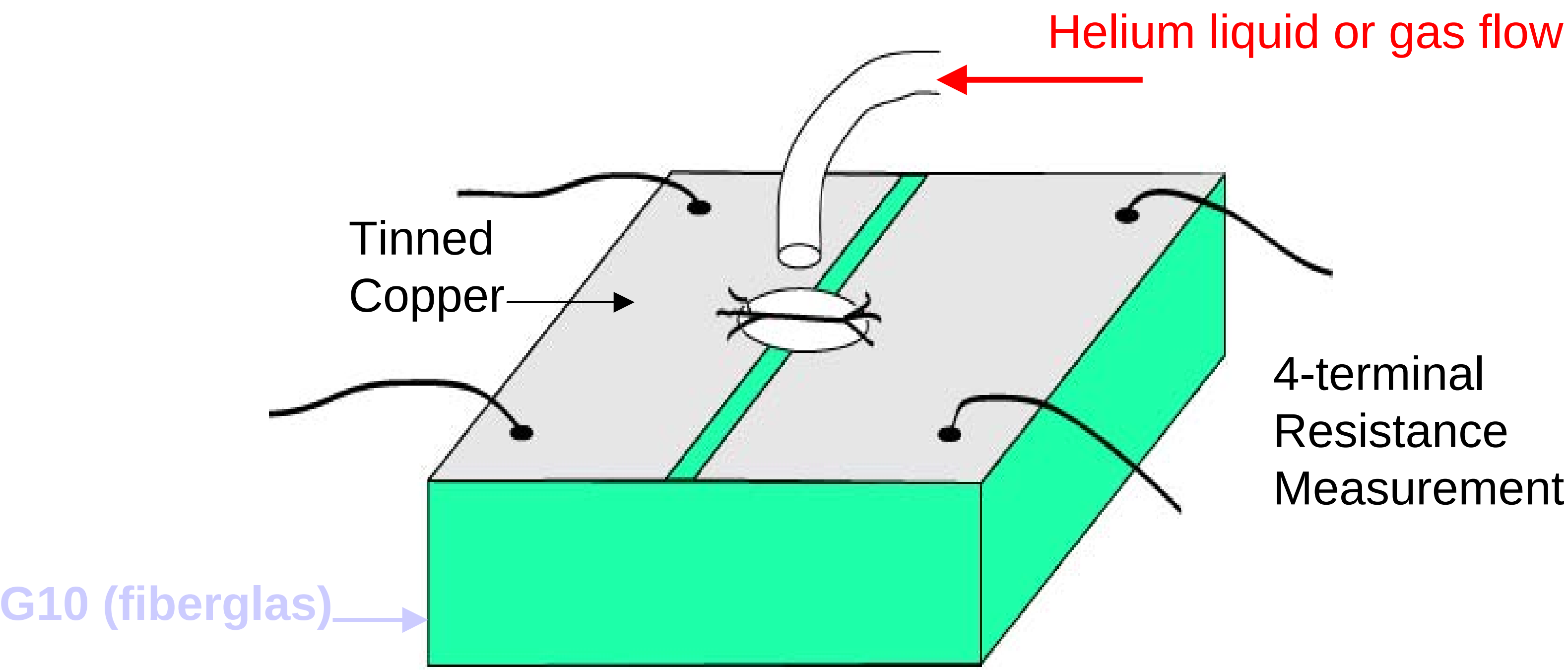
Nanotube Film
R vs. T measurements



Nanotube Flow Meter



Nanotube Film Flow Sensor Test

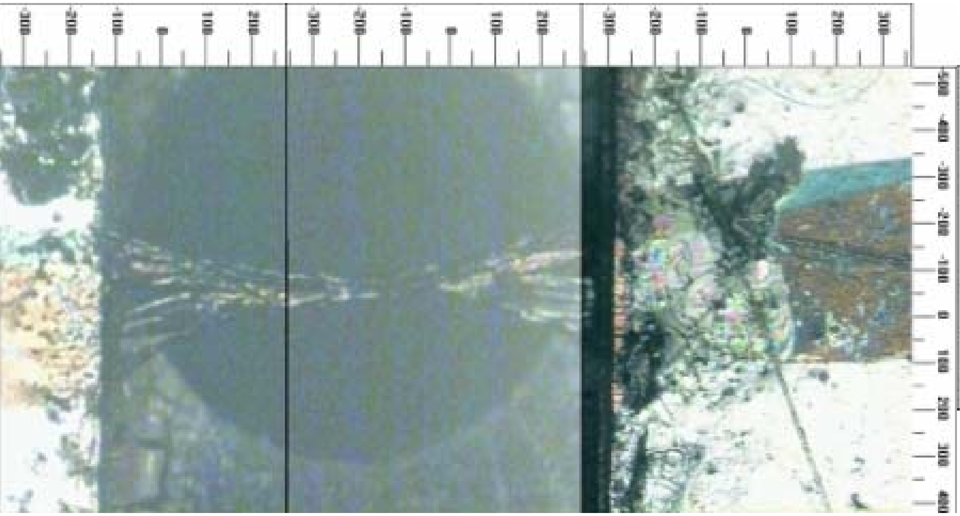


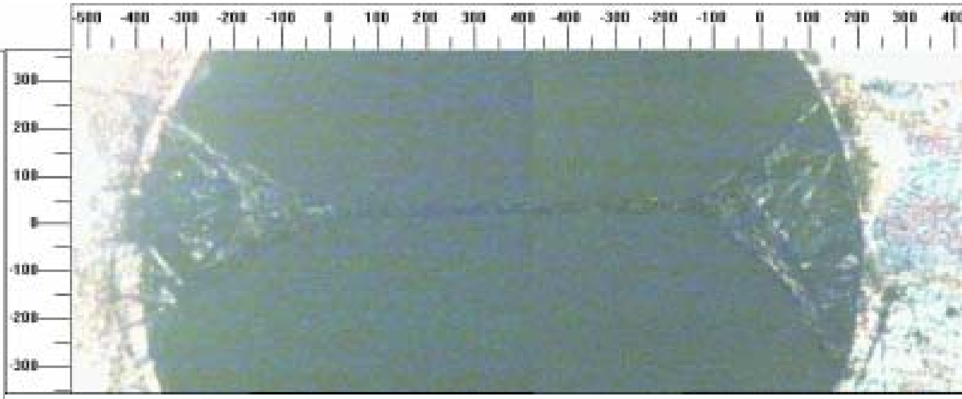
Helium liquid or gas flow

Tinned Copper

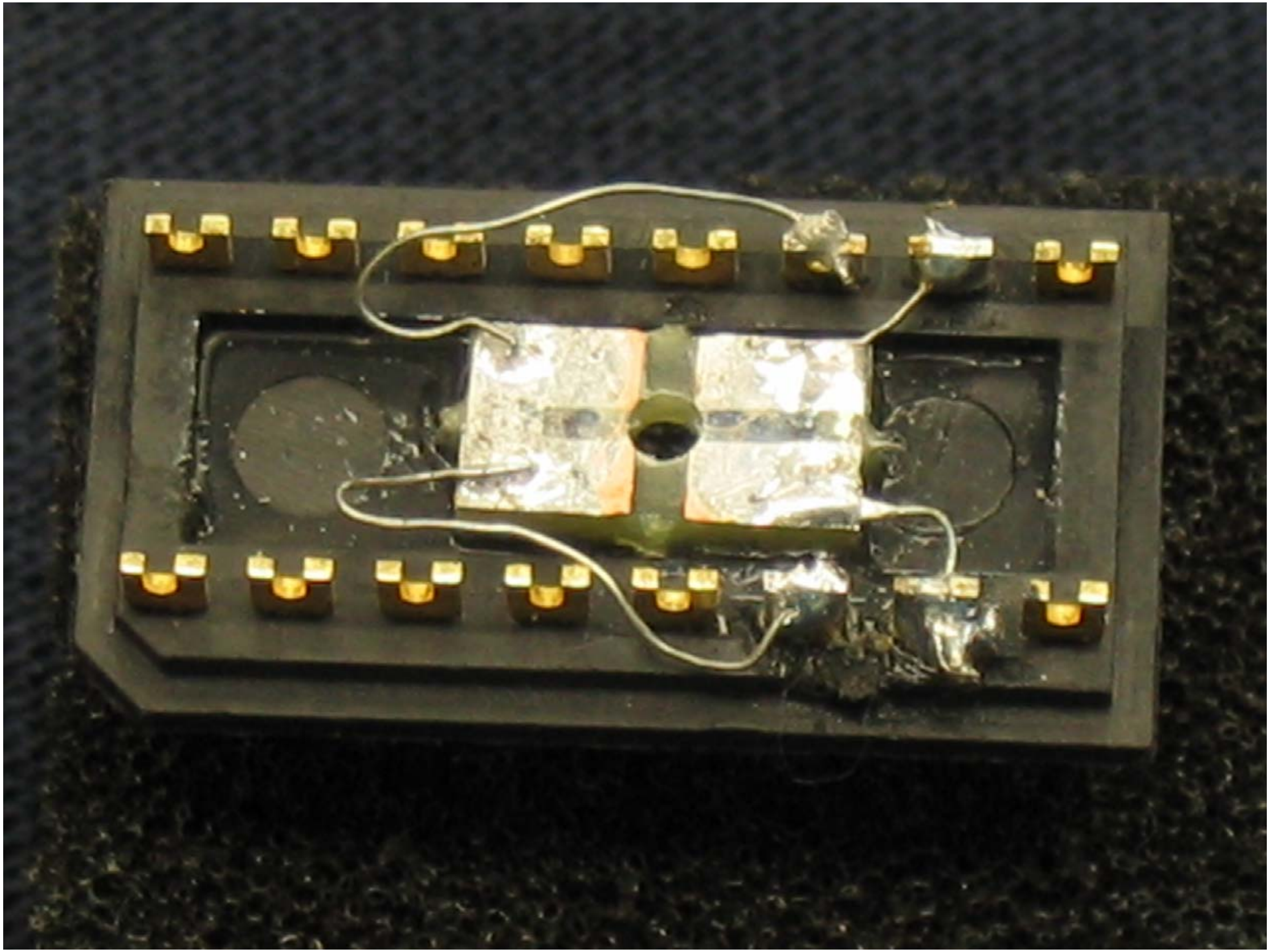
4-terminal Resistance Measurement

G10 (fiberglas)

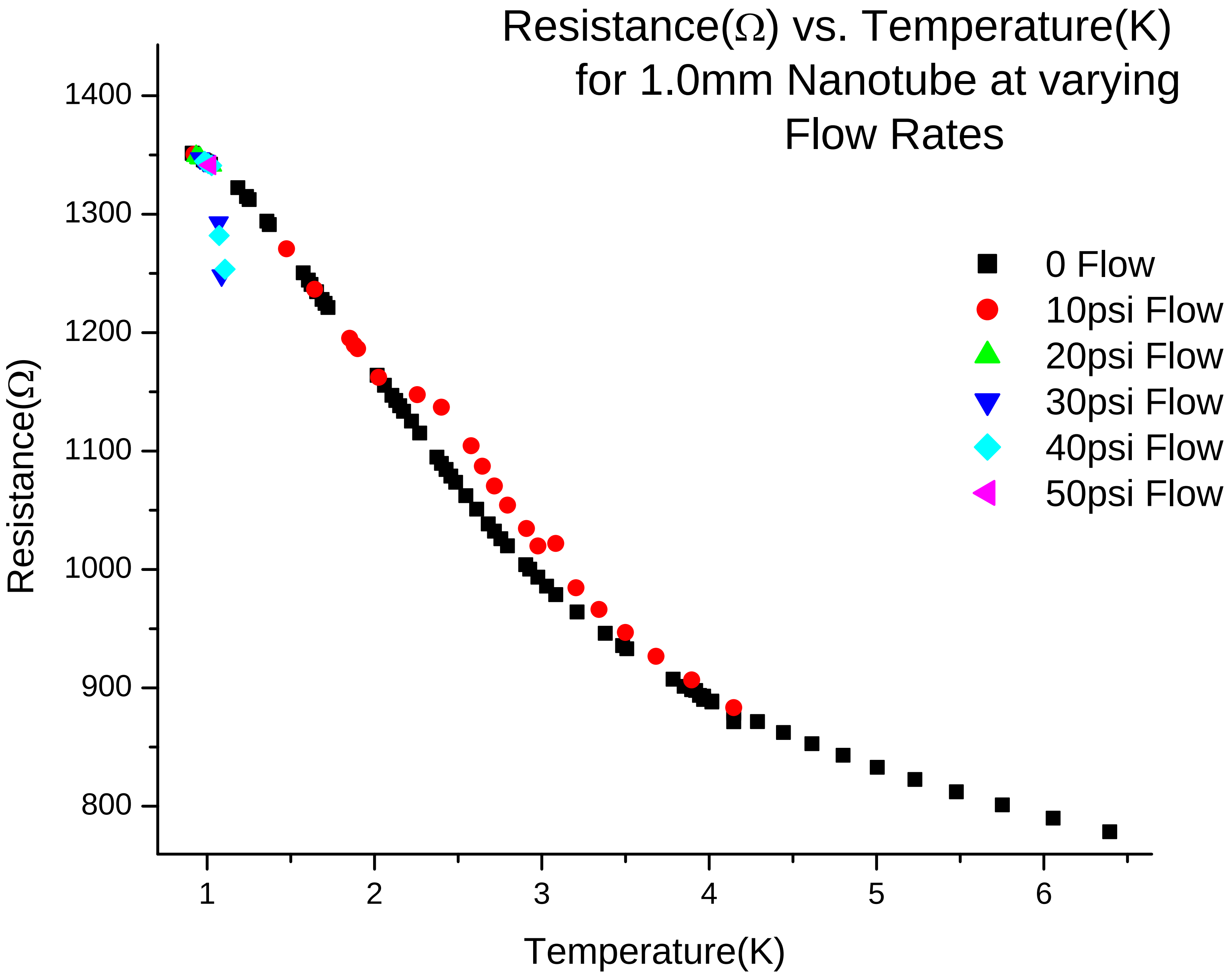




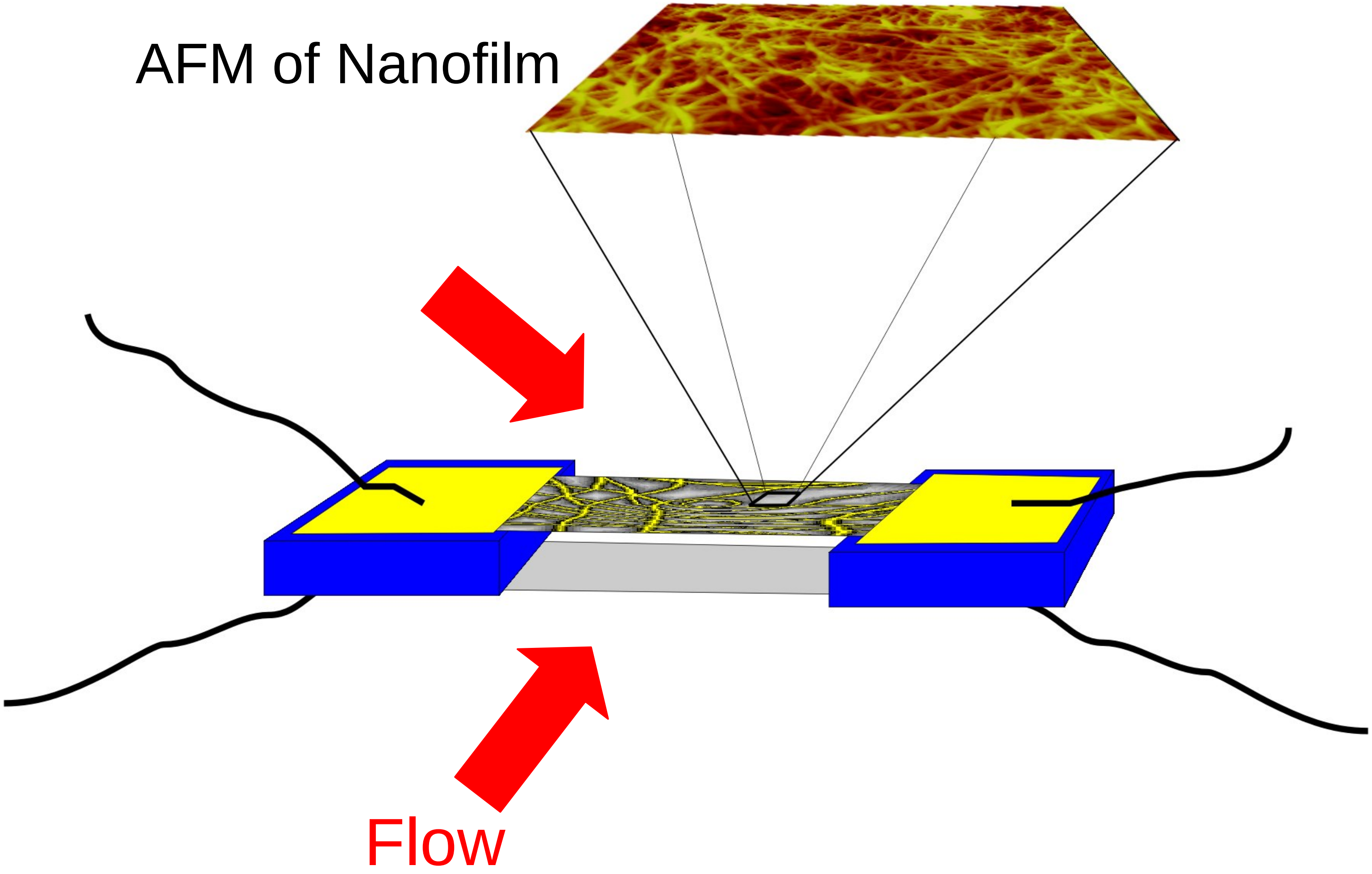
Nanotube film “rope” test jig



Nanotube Flow Meter

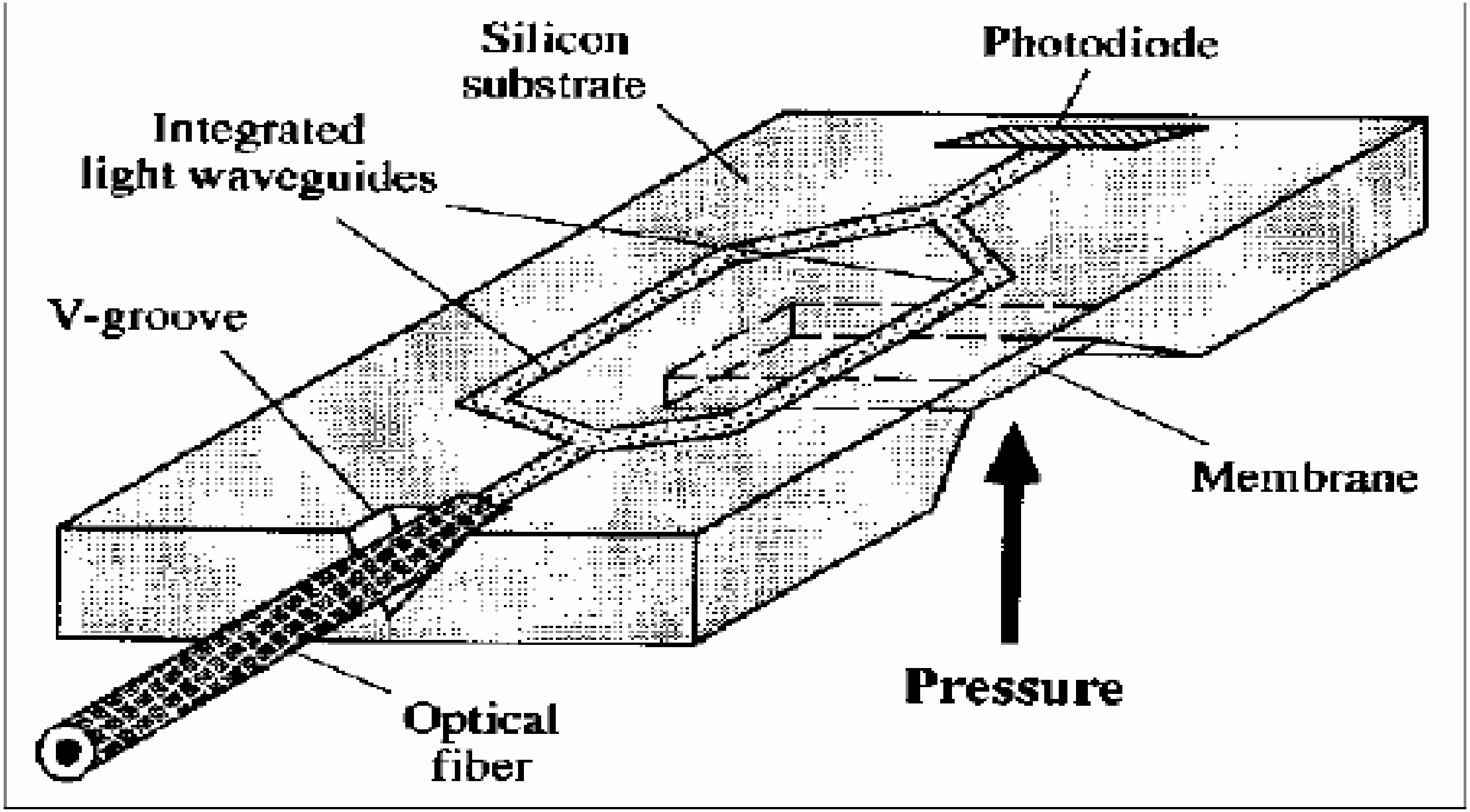


Nanofilm Capacitive Flow/Pressure Fluctuation Sensor



Optically Transduced Pressure Sensors

microsensor structure that deforms under pressure
resulting in a change in an optical signal

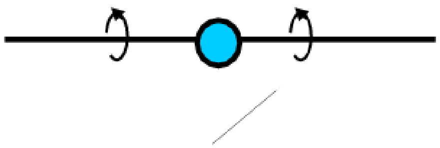


Requirements of PIV in quantum turbulence at low T

- Choice of particle: neutrally-buoyant --helium has a very low density

  small < 1 μm

  bound to vortex line



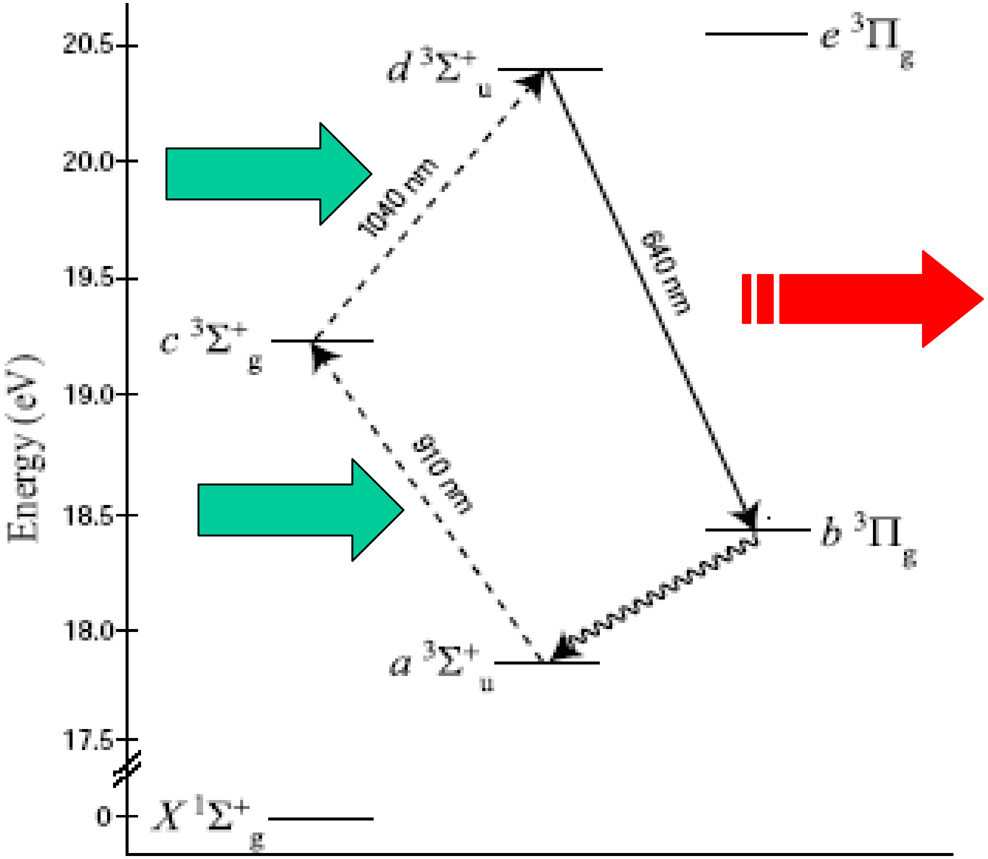
  Type of turbulence: Pulled Grid—Van Sciver got weird results with counterflow

- A new candidate: triplet state He_2 excimer molecules

  $He_2(a^3\Sigma_u^+)$ (McKinsey *et al*, PRL 95, 111101 (2005)).

  lifetime of about 13 s -- radius 0.53 nm -- Production ~ 13000 per Mev.

How does it work?



Illuminate with crossed pulsed lasers at 910 nm and 1040 nm(modest power). Only molecules in crossed beams react

Observe decay of $d^3\Sigma^+_u$ to $b^3\Pi_g$ with emission at 640 nm (lifetime 25 ns).

The $b^3\Pi_g$ returns to $a^3\,\Sigma^+_u$ by non-radiative processes (may need to be accelerated by optical means)

Process recycles.

→ ~ $4\text{x}10^7$ photons/s at 640 nm.

Conclusions

- Important to do grid turbulence measurements at low T in ^{4}He
- Possible to make micron-scale probes of T, P, v?
- Visualization possible
- All depends on much preparatory work = stable budgets for period of years

Thanks for listening!