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Resonant absorption of short pulses

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Abstract

The resonant transformation of a Gaussian pulse of electromagnetic radiation into a Langmuir wave in an inhomogeneous plasma is studied. It is shown that if the pulse duration is smaller than the Langmuir wave (plasmon) lifetime, then substantial changes in the Langmuir field profile occur. Namely, the width of this profile becomes greater and its maximum is smaller compared to the correspondent values for the case of long pulses and the same value of the plasmon lifetime. This fact may be useful for experimental measurements of the electron collision frequency. Indirectly, the collision frequency can be obtained by measuring the intensity of the the second harmonic of the postpulse emission, the expression for which in terms of the Langmuir field is given in this Letter. The limits of validity of linear theory due to plasma profile distortion and wavebreaking are discussed.

The transformation of an electromagnetic wave into Langmuir oscillations in an inhomogeneous plasma (resonant absorption) has been the subject of numerous studies (see Refs. [1,2] and references therein). The physical pattern of the resonant absorption can be summarized as follows. The P-polarized electromagnetic wave, incident on the plasma at an angle θ with respect to the density gradient, has a reflection point at $n = n_c \cos\theta$, $\omega_p(n_c) = \omega_0$. The wave tunnels to the conversion point, $n = n_c$, and resonantly drives Langmuir oscillations with frequency $\omega = \omega_p$. Within the linear theory, the plasma wave amplitude is limited by collisional damping or convection from the resonant zone. The amplitude of the Langmuir field can be much larger than the incident-wave amplitude and have the form of a resonant distribution with a small width (determined, again, by the convection and/or collisions).

The situation can be different for short electromagnetic pulses. The different frequency components contained in the pulse have their conversion points at different locations. As a result, the width of the resonance and the structure of the field within the resonant zone will depend on pulse duration as well. The goal of the present Letter is to study this effect. We will examine the phenomenon of short-pulse resonant absorption in the linear approximation. We will also show how nonlinear processes, such as second harmonic generation, are modified by the finite duration of the pulse.

The problem we address is becoming very important in applications. Advances in laser pulse compression make superhigh laser intensities possible in pulses only a few hundred light wavelengths wide [3]. For experiments with such pulses, resonant absorption is believed to be the major process in laser–plasma interactions. Both finite pulse duration and nonlinear ef-

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fects are important in this process. Even shorter pulses, in terms of the number of wave periods, are available in the microwave frequency range. For example, in the new diagnostic method of plasma reflectometry, the pulse duration can be of the order of ten wave periods [4]. We will show that nonlinear processes can provide valuable information about the plasma, such as frequency of electron collisions.

Let us consider an electromagnetic wave incident on the plasma layer at an angle θ. The longitudinal electric field generated by the monochromatic incident wave with amplitude $E_{0\omega}$ is [1,2]

$$E_z = \frac{\mathrm{i}\sin\theta\, E_{0\omega} A \exp(-\mathrm{i}\omega_0 t)}{z/L - 1 + \omega_p^2/\omega_0^2 + \mathrm{i}\nu/\omega_0}, \tag{1}$$

where ν is the frequency of electron collisions and L is the plasma density length scale. For simplicity, we consider the situation where the Langmuir wave amplitude is limited by collisions. The factor A is proportional to the magnetic field amplitude of the incident wave at the resonant point. It describes the tunneling of the electromagnetic wave to the resonant point and can be expressed in terms of Airy functions [5]. A reasonably good approximation [1,2] is given by $A(q) = 2.3\sqrt{q}\exp(-\frac{2}{3}q^2)$, where $q = (k_0 L)^{2/3}\sin^2\theta$; $k_0 = \omega_0/c$. The function $A(q)$ has a maximum at $q = 0.64$ with width $\Delta q \sim q$, so that $\sin^2\theta \approx 0.64(k_0 L)^{-2/3}$, $|A| \approx 1.2$.

When the convection of Langmuir waves dominates over dissipation, one may still use (1) for estimates with ν replaced by $\nu_{\text{eff}} = \omega_p (r_d/L)^{2/3}$ [2–5]. Hereafter, we will consider only the case $\nu > \nu_{\text{eff}}$, however, most of our results are valid also when the opposite inequality holds if ν replaced by ν_{eff}.

Suppose now that the signal e.m. wave has the form of a Gaussian pulse,

$$E_{\text{signal}} = E_0 \exp(-\mathrm{i}\omega_0 t) = A_0 \exp(-t^2/2\tau^2 - \mathrm{i}\omega_0 t). \tag{2}$$

Since our problem is so far linear, we can use (1) for each Fourier component of the pulse (2) and integrate over all the harmonics according to the superposition principle. Note that different harmonics will have different resonant points, and this will result in a widening of the resonance with corresponding reduction in the peak amplitude.

Taking into account that the Fourier transform of (2) is

$$E_\omega = A_0 \tau \exp[-\tfrac{1}{2}(\omega - \omega_0)^2 \tau^2],$$

we arrive at the expression for the electric field of the Langmuir oscillations,

$$E_z = \int E_{z\omega}\,\mathrm{d}\omega = \mathrm{i}A_0\tau\sin\theta \times \int \frac{A\exp[-\mathrm{i}\omega t - \frac{1}{2}(\omega-\omega_0)^2\tau^2]}{z/L - 1 + \omega_0^2/\omega^2 + \mathrm{i}\nu/\omega}\,\mathrm{d}\omega. \tag{3}$$

Suppose that both ν and τ^{-1} are small compared to ω_0. Then, the integrand of (3) will have a highly pronounced resonance at $\omega = \omega_0$, so that we can disregard the frequency dependence of the factor A (which is a smooth function of ω) and simply take its value at the conversion point. By straightforward algebraic manipulations we can reduce (3) to the form

$$E_z = \tfrac{1}{2}\mathrm{i}\pi^{1/2}\omega_0\tau A_0 \sin\theta\, A(\omega_0) \times \exp(-t^2/2\tau^2 - \mathrm{i}\omega_0 t) Z(\eta), \tag{4}$$

where

$$Z(x) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{\exp(-\lambda^2)\,\mathrm{d}\lambda}{\lambda - x}, \tag{5}$$

and

$$\eta = -\frac{z\omega_0\tau}{2\sqrt{2}L} - \frac{\mathrm{i}t}{\sqrt{2}\tau} + \frac{\mathrm{i}\nu\tau}{2\sqrt{2}}. \tag{6}$$

As usual for the initial value problems, the integral (5) must be taken along a contour below the pole of the integrand. The exponential factor in (4) mimics the time slope of the signal pulse. However, the original shape is distorted in E_z due to the time dependence of $Z(\eta)$ through η. We point out that the function $Z(\eta)$ coincides with the plasma dispersion function which appears in the theory of plasma stability [6].

The finite pulse duration (or finite width of the frequency spectrum) affects resonant absorption in two different ways. First, the frequency variation changes the tunneling conditions (i.e., changes the factor A). On the other hand, for a significant change of A, one must have $\Delta\omega \sim \omega$, i.e., the duration τ must be as small as the wave period (keeping in mind that $\tau \sim$

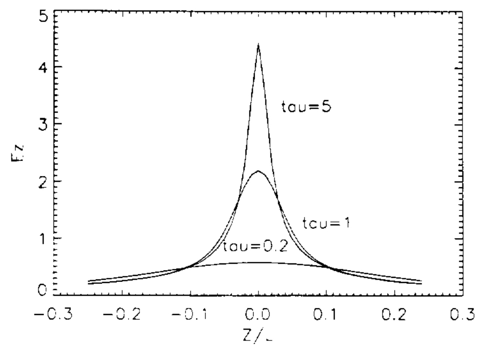

Fig. 1. The spatial profiles of the longitudinal electric field E_z for several different values of the pulse duration τ at time $t = 0$. The carrier wave frequency and the frequency of collisions are $\omega = 100$ and $\nu = 1$, respectively.

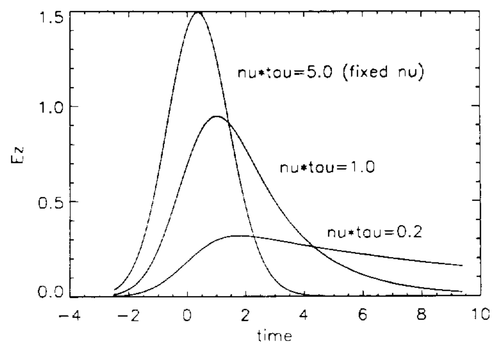

Fig. 2. The the evolution of the longitudinal electric field E_z at the conversion point $z = 0$ for several pulses of different duration and collision frequency $\nu = 1$.

$1/\Delta\omega$ and the wave period $T = 2\pi/\omega$). This effect is not present in (4) because we assumed $\omega\tau \gg 1$. More important is the second effect, namely the displacement of the conversion point and the consequent change of the pulse structure. One can see from (1) that this effect comes into play at $\Delta\omega/\omega = 1/\omega\tau \sim \nu/\omega \ll 1$.

Let us analyze the long-pulse limit, $\nu\tau \gg 1$. Using the asymptotic expansion of Z for large arguments, $Z(\eta) \approx -1/\eta$ [6], we arrive at

$$E_z \approx A_0 \frac{\mathrm{i}\sin\theta\, A \exp(-t^2/2\tau^2 - \mathrm{i}\omega_0 t)}{z/L + \mathrm{i}\nu/\omega_0}. \tag{7}$$

Observe that (7) could be obtained by substitution of (2) into (1) (we measure z such that $\omega_p(0) = \omega_0$). Therefore, the long pulses may be treated as quasi-monochromatic, and one can use directly the results of Refs. [1,2,5]. The maximum value of the electric field is located at $z = 0$ where

$$E_{max} \sim E_0\omega/\nu. \tag{8}$$

The width of the electric field peak is

$$\Delta z \sim L\nu/\omega. \tag{9}$$

In Fig. 1 we present spatial profiles of E_z for several different $\nu\tau$ at $t = 0$. One can see that the maximum of the field is located at $z = 0$, and the profile is symmetric. At the conversion point, $z = 0$, the argument η is purely imaginary and, therefore, we use the following expression for the Z-function,

$$Z(\eta) \approx \mathrm{i}\pi^{1/2}\exp(-\eta^2) - 2\eta \approx \mathrm{i}\pi^{1/2}. \tag{10}$$

According to this expression, in the case $\nu\tau \ll 1$, the Langmuir electric field tends to a value independent of ν and is $\omega_0\tau$ times greater than the amplitude of the incident wave, E_0. This result is natural from the physical point of view. For short pulses, $\nu\tau \ll 1$, the spectral width $\delta\omega \sim 1/\tau$ exceeds the width of the resonant zone, $\Delta\omega \sim \nu$, and only a small fraction, $\nu/\delta\omega \sim \nu\tau$, of the pulse field drives the Langmuir wave. Therefore, E_0 in (8) must be replaced by $E_0\,\nu\tau$, so that

$$E_{z\,max} = E_0\omega_0\tau. \tag{11}$$

For the same reason, the width of the electric field peak is now broader than in the case $\nu\tau \gg 1$,

$$\Delta z \sim L/\omega\tau. \tag{12}$$

In Fig. 2 we show the evolution of E_z at the conversion point, $z = 0$, for several pulses of different duration, as calculated from (4). One can see that, due to small damping, $\nu\tau \ll 1$, Langmuir waves continue to exist long after the pulse duration.

It is remarkable that the total energy absorption in the process of the linear transformation,

$$Q = \nu \int \frac{|E|^2}{8\pi}\,\mathrm{d}z\,\mathrm{d}t, \tag{13}$$

is independent of ν, similar to the case of a monochromatic wave. Indeed, each Fourier component of the electric field contributes to Q independently of all other harmonics, as seen from

$$Q = \nu \int \frac{|E|^2}{8\pi}\, dz\, dt = \nu \int \frac{|E_\omega|^2}{8\pi}\, dz\, d\omega. \qquad (14)$$

Contribution of individual harmonics is ν-independent and so is the integral over all the harmonics contributing to Q.

Thus, the total efficiency of the energy absorption is independent of the pulse duration. In contrast, the pulse shortening essentially affects various nonlinear processes, e.g., second harmonic generation accompanying the resonant transformation of the electromagnetic wave. Indeed, due to the nonlinearity of the Maxwell equations coupled with the plasma equations and the high intensity of the electromagnetic wave, one will observe a strong emission at $\omega \approx 2\omega_0$ [7]. The intensity of this emission is given by the following expression [7],

$$I = \frac{c}{32\pi k_2^2} \left| \int_{-\infty}^{\infty} F(z) \exp(-ik_2 z)\, dz \right|^2, \qquad (15)$$

where the "source" of the second harmonics has the form

$$F = \frac{ie}{2mc\omega} \frac{d}{dz} \left(i\frac{\omega}{c} \sin\theta\, E_z^2 + E_y \frac{d}{dz} E_z \right),$$

$$k_2^2 = \frac{4\omega^2}{c^2} \left(\tfrac{3}{4} - \sin^2\theta \right).$$

The first term in F can be interpreted as the creation of a 2ω-quantum via the merging of two Langmuir waves (plasmons), and the second term as the creation of such a quantum through the merging of the plasmon and the electromagnetic wave. In the case of long pulses, the second term will dominate. For short pulses, the second harmonic will be weaker due to the drop in the electric field. In this case, the second term will vanish after the pulse terminates. On the other hand, second harmonic generation via the two-plasmon merging will persist for a longer time, because of the finite life time of the Langmuir waves after the pulse termination. Thus, detection of second harmonic radiation can provide a direct measurement of plasmon damping, and whatever process is responsible for it (i.e., collisions, wavebreaking, etc.).

Similar behavior is manifested in electromagnetic radiation with the frequency of the incident light – the process inverse to the resonance transformation. For short pulses this process lasts until the plasmons decay, so that the monitoring of this "postpulse" radiation also can provide a measurement of the Langmuir wave damping near the resonant layer.

To this point, we have discussed the effect of the pulse finiteness in time. The pulse may also have some transverse spatial structure, for example, "hot spots". Formally, it would mean that the pulse is composed of harmonics with the same frequencies, but different wavenumbers or angles of incidence, θ. One can see from (1) that the transverse dependence will modify the field structure only through Λ, which is dependent on $q(k,\theta)$. Such a dependence is very weak, and, therefore, the resonant absorption is much less sensitive to spatial variations than to temporal ones.

There are some restrictions on the applicability of the linear theory of resonant absorption just presented. The resonantly excited Langmuir wave creates strong ponderomotive forces, deforming the plasma density profile near the resonance point. When this density modification, δn, becomes comparable with the density variation of the plasma profile on the width of the resonance, $\Delta z \sim L\nu/\omega$, then the linear theory breaks down. In this case, scale splitting and transition to turbulence occur [8,9]. In the long-pulse case, we have the following criterion of applicability of the linear theory [10],

$$\frac{\delta n}{n} \approx \frac{E_1^2}{8\pi nT} \approx \frac{E_0^2}{8\pi nT} \frac{\omega^2}{\nu^2} < \frac{\Delta z}{L} \sim \frac{\nu}{\omega}.$$

For short pulses, the density depletion is determined by the ion inertia rather than the pressure balance. As a result, the density variation is smaller than $E_1^2/8\pi nT$ by the factor of $(c_s\tau)^2/(\Delta z)^2$ if $c_s\tau < \Delta z$, where c_s is the speed of sound. Also, the density variation drops due to the decrease of the maximum electric field, see (11). For such a situation

$$\frac{\delta n}{n} \approx \frac{E_0^2}{16\pi nT} (\omega\tau)^4 \frac{\tau^2 c_s^2}{L^2}.$$

The linear theory is applicable in this case up to

$$\frac{E_0^2}{16\pi nT} < \frac{M}{m} \frac{L^2}{r_D^2} \frac{1}{(\omega\tau)^7},$$

where r_D is the Debye radius. The range of validity rapidly expands with decrease of τ.

Other factors, which limit applicability of the linear theory are electron nonlinearities and the Langmuir wavebreaking. If high intensity pulsed laser radiation is used, then wavebreaking can occur faster than the nonlinear profile deformation by the radiative force. The time of wavebreaking τ_w is given by the following estimate [11],

$$\tau_w \sim \sqrt{Lm/eE_z}.$$

One can see that for short pulses, the wavebreaking time increases due to the drop in the electric field [9]. If the pulse duration is smaller than τ_w, the linear theory can be used for the description of the resonant absorption process (provided that the nonlinear profile deformation does not occur). Note, however, that the subsequent evolution of the Langmuir waves can be essentially nonlinear.

Thus, both limitations, associated with the nonlinear profile deformation and wavebreaking, are less restrictive in the case of short pulses; and the linear theory, developed in this Letter, can be used for much more powerful pulses, than in the long-pulse regime.

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