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Non-local MHD turbulence

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Abstract

We consider an example of strongly non-local interaction in incompressible magnetohydrodynamic (MHD) turbulence which corresponds to the case where the Alfvén waves travelling in the opposite directions have essentially different characteristic wavelengths. We use two approaches to the dynamics of turbulent Alfvénic wavepackets: the first is a geometrical WKB theory [Phys. Lett. A 165 (1992) 330] and the second one is a three-wave kinetic equation derived for weakly turbulent waves [J. Plasma Phys., in press]. We show that these theories have a common limit of weak turbulence with scale separation in which they both predict the same Fokker–Planck equation for the wave power spectrum. In both cases the packet wavenumbers (and therefore the Lagrangian field-line separations) are allowed to experience order 1 changes. The WKB theory developed here formalises an intuitive geometrical argument of Goldreich and Sridhar [ApJ 485 (1997) 680] and allows one to see where such an intuition leads to a wrong conclusion about the inapplicability of the three-wave kinetic equation for order 1 wavepacket distortions. We show that the exponent of the constant flux non-local spectrum matches the value previously found for local turbulence at the boundary of the locality interval. The relationship between the WKB theory and the weak turbulence theory found in this paper for an ensemble of Alfvén waves seems to be general for three-wave systems.

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1. Introduction

Considering the special theme of this Physica D issue, we take the opportunity to start off by telling a story about one of many remarkable ideas generated in Vladimir Zakharov's school and which have materialised themselves later into exciting research areas. This is a story about the non-local turbulence and it begins when the newly found and promising Kolmogorov spectra of weak drift and Rossby wave turbulence were tested for locality and stability [2]. The result was negative: all the spectra turned out to be either unstable or non-local, some of them in a quite peculiar and nontrivial way (e.g., the non-locality developing in time from small perturbations [2,3]), but with the same consequences regarding their realisability. In short, they were irrelevant for any further study of the Rossby wave turbulence. On the positive side, it was noticed, however, that the typical way in which non-locality manifests itself is an infra-red (large scale) divergence of the collisional integral of the kinetic equation. In other words, the nonlocal exchange of energy via the largest scales of motion far overweighs the local interaction of smaller

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scales among themselves. This fact is intimately related with the real physical effect of energy condensation at large scales via inverse cascade processes, which is typical not only to the Rossby dynamics but also to a much broader class of two-dimensional (2D) systems including 2D Navier–Stokes turbulence. This observation was very useful because it allowed one to formulate the nonlocality hypothesis for Rossby turbulence which provides an alternative to the Kolmogorov picture of local turbulence and which suggests that one should only retain the wavenumber triads which have one leg much shorter than the two others. The nonlocality hypothesis allows one to reduce the integro-differential wave kinetic equation to a much simpler differential Fokker–Plank (FP) equation and to describe the saturation mechanism for the large-scale energy condensation accompanied by the formation of a jet-like spectrum at small scales and a spectral gap between [4,21]. Further, the nonlocality hypothesis allows one to by-pass the weak turbulence assumptions and apply the scale separation technique directly to the dynamical equations [1]. This way, there is no need to average over random large scales and one can deal with condensates having the form of coherent vortices. The outcome of such an approach is a coupled system of equations for the large and small scales which is a complete system for describing nonlocal turbulence. It was also noticed that the small-scale equation (which is in waveaction conservation law form) yields the same FP equation as the kinetic equation does if the large scales are random [17]. The concept of turbulence nonlocality introduced in [1,2,4,21] and corresponding technique of the scale separation has recently found numerous applications such as the acoustic–vortex interactions [5], interaction of waves with zonal flows in polar stratospheres [6], Taylor four-roller mill experiment [7], near-wall turbulence [8], atmospheric boundary layer [10], transport coefficients in magnetic fluids [11] and the sub-grid scale modelling [9]. In this paper we apply this technique to consider nonlocal magnetohydrodynamic (MHD) turbulence with oppositely propagating Alfvén waves having very different characteristic scales. Such strongly anisotropic and asymmetric situations are not uncommon in astrophysical applications. However, an additional motivation to study this situation arose from a recent controversy which is described below.

There has been a debate in the recent literature about the applicability of weak turbulence theory based on the three-wave process to Alfvén wave ensembles in incompressible magnetohydrodynamic (MHD). Goldreich and Sridhar (hereafter GS) [19] claimed that the three-wave interaction is empty. After criticisms by Montgomery and Matthaeus [15] and Ng and Bhattachargee [18], GS recognised that this was wrong but they put forward a new geometrical argument to show that the four-wave and higher order processes are as important as three-wave ones even if the wave amplitudes are small [14]. Their construction presented in Section 2.2 of [14] considers an Alfvénic wavepacket propagating along a stochastic bundle of magnetic field lines whose deviation from the uniform field b_0 is produced by counter-propagating Alfvén waves. They argue correctly that the energy transfer length scale L_* is determined by the distance at which the separation between neighbouring field lines increases by a factor of order unity from its initial value. Indeed, after this time the packet wavenumbers experience order 1 changes and so does the energy spectrum at fixed wavenumber (because the total energy is conserved). Further, GS correctly estimated this transfer distance to be ϵ^{-2}, where ϵ is the nonlinearity parameter (χ in [14]). Then, they identify the wavepacket amplitude changes corresponding to the n-wave process with consecutive Taylor expansion terms in $(l/L_*)^{1/2}$ where l is the travelled distance (which, presumably, has to be squared to get the power spectrum). The immediate conclusion from this statement is that the contribution of four-wave and higher processes becomes comparable to the three-wave contribution for $l = L_*$, i.e., for the time needed for a non-negligible energy transfer. However, this statement is incorrect. Indeed, as it is well understood in classical weak turbulence theory [12,20], the three-wave process contains all the infinite number of terms $\sim (l/L_*)^j$, $j = 1, 2, 3, \dots$. On the other hand, the n-wave process is made of terms $\sim (l\lambda^{n-3}/L_*^{n-2})^j$, $j = 1, 2, 3, \dots$, where λ is the initial packet wavelength (i.e., the energy injection scale). An explanation of this fact, although well documented, is quite lengthy and too technical for non-specialists. Therefore, we choose another strategy and present a simple counter-example to GS's argument by deriving a mathematical formalism for their own physical set-up and by showing that its predictions coincide with the ones derived within the three-wave weak turbulence approach for the time of order of the energy transfer time.

Besides giving a counter-example to the GS's argument, this paper presents an example of an important relationship between weak turbulence and the WKB theory which appears to be quite general. Namely, the case when one of the interacting three waves is always longer than the other two and the wave amplitudes are small can be considered using two equivalent approaches. The first of these approaches, WKB, considers a linear equation for short-wave packets propagating on the slowly varying background produced by the long wave. The second approach, weak turbulence theory, deals with a wave kinetic equation based on the three-wave resonant interactions. We show that, for Alfvén waves, both of these approaches lead to the same (FP) equation for the turbulence spectrum.

2. WKB theory

To derive a mathematical formalism for the motion of Alfvénic wavepackets, we consider a special case when the waves propagating along the magnetic field are much shorter than the oppositely propagating Alfvén waves. For the Elsässer variables $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ ($\mathbf{v}$ and $\mathbf{b}$ are the velocity field and the magnetic field, respectively), we have $\mathbf{z}^{+} = \mathbf{z}_{\mathrm{S}}^{+} + \mathbf{B}_0$ and $\mathbf{z}^{-} = \mathbf{z}_{\mathrm{L}}^{-} - \mathbf{B}_0$. Here subscripts L and S denote large and small scales respectively. For $\mathbf{z}^{+}$ we have the following linear equation:

$$(\partial_t + B_0\nabla_{\parallel} + \mathbf{z}_{\mathrm{L}}^{-} \cdot \nabla)\mathbf{z}_{\mathrm{S}}^{+} = -\nabla P, \tag{1}$$

where P is the pressure.

The scale separation allows us to develop a WKB theory for small-scale wavepackets. This can be done using the Gabor transform defined as

$$\hat{\mathbf{z}}^{+}(\mathbf{x}, \mathbf{k}, t) = \int g(\sigma|\mathbf{x} - \mathbf{x}'|)\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\mathbf{z}_{\mathrm{S}}^{+}(\mathbf{x}', t)\, \mathrm{d}\mathbf{x}', \tag{2}$$

where $1 \gg \sigma \gg l_{\mathrm{S}}/l_{\mathrm{L}}$ (l_{S} and l_{L} are the small and large length scales, respectively) and g is a function which rapidly decreases at infinity. For details of using the Gabor transform for WKB theory see [7,8,16]. Applying the Gabor transform to (1), we have

$$\partial_t\hat{\mathbf{z}}^{+} + (\mathbf{z}_{\mathrm{L}}^{-} \cdot \nabla + B_0\nabla_{\parallel})\hat{\mathbf{z}}^{+} - \nabla(\mathbf{k} \cdot \mathbf{z}_{\mathrm{L}}^{-}) \cdot \nabla_{\mathbf{k}}\hat{\mathbf{z}}^{+} = -\mathrm{i}\mathbf{k}\hat{P}. \tag{3}$$

The LHS of this equation can be written as $D_t\hat{\mathbf{z}}^{+}$, where $D_t = \partial_t + \dot{\mathbf{x}} \cdot \nabla + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}}$ is a time derivative along the wave rays given by $\dot{\mathbf{x}} = \nabla_{\mathbf{k}}\omega$, $\dot{\mathbf{k}} = -\nabla\omega$, with ω being the Doppler shifted frequencies of the positively propagating Alfvénic wavepackets, $\omega = B_0k_{\parallel} + \mathbf{k} \cdot \mathbf{z}_{\mathrm{L}}^{-}$. The expression for the pressure $\hat{P}$ can be found by multiplying the relation (3) by $\mathbf{k}$ and by taking into account the divergence-free condition $\mathbf{k} \cdot \hat{\mathbf{z}}^{\pm} = 0$. This leads to the expression (compare with [8])

$$-\mathrm{i}k^2\hat{P} = \hat{\mathbf{z}}^{+} \cdot \nabla\omega. \tag{4}$$

Substitution of this relation into (3) gives

$$D_t\hat{\mathbf{z}}^{+} = \frac{\mathbf{k}}{k^2}\hat{\mathbf{z}}^{+} \cdot \nabla\omega. \tag{5}$$

Multiply this expression by $(\hat{\mathbf{z}}^{+})^{*}$ ($*$ stands for the complex conjugate) and take the real part of the resulting equation. We have

$$D_te^{+} = \partial_te^{+} + \nabla \cdot (\dot{\mathbf{x}}e^{+}) + \nabla_k \cdot (\dot{\mathbf{k}}e^{+}) = 0, \tag{6}$$

where the wave action spectrum $e^+(\mathbf{x}, \mathbf{k}) = \hat{\mathbf{z}}^+\hat{\mathbf{z}}^{+*}$. Eq. (6) is the main equation of the geometrical WKB theory which is sometimes referred to as the transport equation. It describes conservation of wave action along the trajectories in coordinate-wavenumber space.

3. Averaging over the large scales

Let us assume now that the large-scale motions z_L are random and that they are weak compared to the mean magnetic field, $z_L \ll B_0$. This means that the wavepackets $\hat{\mathbf{z}}^+$ will move with almost constant speed along the axis $\hat{\mathbf{e}}_\parallel$ and experience stochastic changes in $\mathbf{k}$ as they travel through $\mathbf{z}_L^-$. The spectrum e^+ consists of a strong spatially uniform component $e^+_{(0)}(\mathbf{k}, t)$ and weak "wiggles" $e^+_{(1)}(\mathbf{x}, \mathbf{k}, t)$ such that $e^+(\mathbf{x}, \mathbf{k}, t) = e^+_{(0)}(\mathbf{k}, t) + e^+_{(1)}(\mathbf{x}, \mathbf{k}, t)$, with $e^+_{(0)} = L^{-N}\int e^+ \mathrm{d}\mathbf{x}$, $\int e^+_{(1)}\,\mathrm{d}\mathbf{x} = 0$, $e^+_{(1)} \ll e^+_{(0)}$. N is the spatial dimension of the problem. Note that integration over the spatial coordinates is equivalent, in this case, to ensemble averaging. With these notations, we have

$$\partial_t e^+_{(0)} = L^{-N}\boldsymbol{\nabla}_k \cdot \int \boldsymbol{\nabla}(z_L^- \cdot \mathbf{k}) e^+_{(1)}\,\mathrm{d}\mathbf{x}. \tag{7}$$

To find $e^+_{(1)}$, we introduce a time scale T such that the correlation time of z_L^- (i.e., the large-scale wave period) is much smaller than T and the characteristic time of $e^+_{(0)}$ ($\sim \epsilon^{-2}$) is much larger than T. By using the conservation of e^+ along the trajectories, we have

$$e^+(\mathbf{x}, \mathbf{k}, t) = e^+_{(0)}\left(\mathbf{k} - \int_0^T \dot{\mathbf{k}}\,\mathrm{d}t, 0\right) + e^+_{(1)}\left(\mathbf{x} - \int_0^T \dot{\mathbf{x}}\,\mathrm{d}t, \mathbf{k} - \int_0^T \dot{\mathbf{k}}\,\mathrm{d}t, 0\right) \tag{8}$$

$$\simeq e^+_{(0)}(\mathbf{k}, 0) - \left(\int_0^T \dot{\mathbf{k}}\,\mathrm{d}t\right) \cdot \boldsymbol{\nabla}_k e^+_{(0)}(\mathbf{k}, 0) + e^+_{(1)}(\mathbf{x} - B_0 T, \mathbf{k}, 0), \tag{9}$$

where all the integrations are along the wave rays. Thus,

$$e^+_{(1)}(\mathbf{x}, \mathbf{k}, t) \simeq -\left(\int_{t-T}^t \dot{\mathbf{k}}\,\mathrm{d}t'\right) \cdot \boldsymbol{\nabla}_k e^+_{(0)}(\mathbf{k}, t) + e^+_{(1)}(\mathbf{x} - B_0 T, \mathbf{k}, t - T). \tag{10}$$

Substituting this relation in (7) and taking into account that the second term in RHS of (10) will not contribute because T is greater than the correlation time, we arrive at the FP equation

$$\partial_t e^+_{(0)} = \boldsymbol{\nabla}_k \hat{D} \boldsymbol{\nabla}_k e^+_{(0)}, \tag{11}$$

where the diffusion coefficient is

$$D_{\alpha\beta} = L^{-N} \int \nabla_\alpha (z_L^-(\mathbf{x}, t) \cdot \mathbf{k}) \int_{t-T}^t \nabla_\beta (\mathbf{z}_L^-(\mathbf{x} - \mathbf{B_0}(t - t'), t') \cdot \mathbf{k})\,\mathrm{d}\mathbf{x}\,\mathrm{d}t'. \tag{12}$$

In the frame moving with speed $-B_0$ (taking into account $z_L^-(\mathbf{x}, t) \approx z_L^-(\mathbf{x} + B_0 t \hat{\mathbf{e}}_\parallel)$), we have the following expression:

$$\begin{aligned} D_{\alpha\beta} &= L^{-N} \int \nabla_\alpha (z_L^-(\mathbf{x}) \cdot \mathbf{k}) \int_{t-T}^t \nabla_\beta (\mathbf{z}_L^-(\mathbf{x} + 2B_0 \hat{\mathbf{e}}_\parallel (t' - t), t') \cdot \mathbf{k})\,\mathrm{d}\mathbf{x}\,\mathrm{d}t' \\ &= L^{-N} \int \kappa_\alpha (\hat{z}_L^-(\kappa) \cdot \mathbf{k}) \int_{t-T}^t \kappa_\beta\, \mathrm{e}^{-\mathrm{i}\kappa_z 2B_0 (t'-t)} (\hat{\mathbf{z}}_L^-(-\kappa) \cdot \mathbf{k})\,\mathrm{d}\kappa\,\mathrm{d}t', \end{aligned} \tag{13}$$

where $\hat{z}_{\rm L}^{-}(\kappa)$ is the Fourier transform of $z_{\rm L}^{-}$. Because $T \gg (\boldsymbol{\kappa}_\perp B_0)^{-1}$, the time integration of $e^{-i\kappa_z 2B_0 t'}$ gives $(\pi/2B_0)\delta(\kappa_z)$. Thus, we finally have

$$D_{\alpha\beta} = \frac{\pi}{2B_0}\int \kappa_\alpha \kappa_\beta F_{ij}^{-}(\boldsymbol{\kappa}_\perp, 0) k_i k_j \, d\kappa_1 \, d\kappa_2, \quad \alpha, \beta = 1, 2, \tag{14}$$

and $D_{\alpha\beta} = 0$ if $\alpha = 3$ or $\beta = 3$. Here $F_{ij}^{-}(\boldsymbol{\kappa}_\perp, \kappa_\parallel) = L^{-N}\hat{\mathbf{z}}_{Li}^{-}(\boldsymbol{\kappa})\hat{\mathbf{z}}_{Lj}^{-}(-\boldsymbol{\kappa})$. The expression for $D_{\alpha\beta}$ reveals the unique contribution of the mode $\kappa_\parallel = 0$ to the diffusion, and therefore the absence of energy transfer along $\mathbf{B}_0$.

4. Weak turbulence theory

We are going to show now that the same FP equation can be derived using the weak turbulence approach based on the three-wave kinetic equation proposed in [13]. This equation is of the form

$$\partial_t e^{+}(\mathbf{k}) = \int G(\mathbf{k}, \boldsymbol{\kappa}, \mathbf{L})\delta(\mathbf{k} - \boldsymbol{\kappa} - \mathbf{L})\delta(\kappa_\parallel) \, d\boldsymbol{\kappa} \, d\mathbf{L}, \tag{15}$$

where the contribution of the nonlinear three-wave interactions $G(\mathbf{k}, \boldsymbol{\kappa}, \mathbf{L})$ is given in [13]. Note that in this case e^{+} means the ensemble averaged turbulence spectrum which is equivalent to e_0^{+}, the coordinate averaged WKB spectrum considered above. The absence of parallel transfer and the catalytic role played by the $\kappa_\parallel = 0$ state arise in the kinetic equation because of $\delta(\kappa_\parallel)$ which, in turn, is a consequence of the three-wave resonant condition. Below, we will be interested in the special case of weak turbulence with scale separation, such that

$$\kappa_\perp \ll k_\perp, L_\perp, \tag{16}$$

which allows us to introduce the notation $\mathbf{L}_\perp = \mathbf{k}_\perp + \ell_\perp$ with $\ell_\perp \ll 1$ and approximate the collision integral by a differential (FP) expression. In this case, we will only need to know G for values where the first and the third arguments are approximately equal. At leading order, we find [13]

$$G(\mathbf{k}, \boldsymbol{\kappa}, \mathbf{k} + \ell_\perp) \simeq \frac{\pi \epsilon^2}{B_0}\left(\ell_\perp \cdot \frac{\partial e^{+}(\mathbf{k})}{\partial \mathbf{k}_\perp}\right) Q_k^{-}(\boldsymbol{\kappa}), \tag{17}$$

where $Q_k^{-}(\boldsymbol{\kappa}) = k_i k_j q_{i\,j}^{--}(\boldsymbol{\kappa})$ and $q_{i\,j}^{--}(\boldsymbol{\kappa})\delta(\boldsymbol{\kappa} + \boldsymbol{\kappa}') = \langle \hat{\mathbf{z}}_i^{-}(\boldsymbol{\kappa})\hat{\mathbf{z}}_j^{-}(\boldsymbol{\kappa}')\rangle$. In this case, it is straightforward to show that this assumption leads to the same FP equation. (Note that the similar conclusion that the WKB and the three-wave kinetic equation have a common limit represented by a FP equation was made before for Rossby waves [17].) Next, let us introduce a smooth function $f(\mathbf{k})$ which decays at the limits of integration then integrate (15) in $\mathbf{k}_\perp$ using the symmetry of the integrand. This gives

$$\begin{aligned}\partial_t \int f(\mathbf{k}) e^{+}(\mathbf{k}) \, d\mathbf{k}_\perp &\simeq \int f(\mathbf{k}) G(\mathbf{k}, \boldsymbol{\kappa}_\perp, \mathbf{k} + \ell_\perp)\delta(\ell_\perp + \boldsymbol{\kappa}_\perp) \, d\boldsymbol{\kappa}_\perp \, d\ell_\perp \, d\mathbf{k}_\perp, \\ &\simeq \frac{1}{2}\int [f(\mathbf{k}) G(\mathbf{k}, \boldsymbol{\kappa}_\perp, \mathbf{k} + \ell_\perp) + f(\mathbf{k} + \ell_\perp) G(\mathbf{k} + \ell_\perp, \boldsymbol{\kappa}_\perp, \mathbf{k})] \\ &\quad \times \delta(\ell_\perp + \boldsymbol{\kappa}_\perp) \, d\boldsymbol{\kappa}_\perp \, d\ell_\perp \mathbf{k}_\perp.\end{aligned} \tag{18}$$

The symmetry $G(\mathbf{k}, \boldsymbol{\kappa}, \mathbf{k} + \ell_\perp) = -G(\mathbf{k} + \ell_\perp, \boldsymbol{\kappa}, \mathbf{k})$, which is the reason for the absence of energy transfer along the mean magnetic field, leads to the expression

$$\partial_t \int f(\mathbf{k}) e^{+}(\mathbf{k}) \, d\mathbf{k}_\perp \simeq \frac{1}{2}\int G(\mathbf{k}, \boldsymbol{\kappa}_\perp, \mathbf{k} + \ell_\perp)\left(-\ell_\perp \cdot \frac{\partial f(\mathbf{k})}{\partial \mathbf{k}_\perp}\right)\delta(\ell_\perp + \boldsymbol{\kappa}_\perp) \, d\boldsymbol{\kappa}_\perp \, d\ell_\perp \mathbf{k}_\perp. \tag{19}$$

If we integrate by parts and use the fact that function $f(\mathbf{k})$ is arbitrary, we obtain

$$\partial_t e^+(\mathbf{k}) \simeq \frac{1}{2}\int \ell_\perp \cdot \frac{\partial G(\mathbf{k}, \boldsymbol{\kappa}_\perp, \mathbf{k}+\ell_\perp)}{\partial \mathbf{k}_\perp}\delta(\ell_\perp + \boldsymbol{\kappa}_\perp)\,\mathrm{d}\boldsymbol{\kappa}_\perp\,\mathrm{d}\ell_\perp. \tag{20}$$

Substituting expression (17) into the previous equation leads to the same FP equation as in (11)

$$\partial_t e^+(\mathbf{k}) = \nabla_k \hat{D} \nabla_k e^+(\mathbf{k}), \tag{21}$$

where the diffusion coefficient $D_{\alpha\beta}$ is exactly the same as (14), and where F^-_{ij} identifies to $q^{--}_{i\,j}$.

5. Conclusion

As we see, both the WKB formalism of GS's geometrical argument and three-wave weak turbulence theory lead to the *same* FP equation for the energy spectrum in the limit of small amplitude waves with scale separation. From our WKB derivation, it is obvious that the packet wavenumbers can experience order 1 changes when they travel through many periods of the oppositely propagating large-scale waves, and such changes correspond to order 1 changes in the energy spectrum described by the FP equation. Thus, the three-wave interaction and the geometrical wave packet propagation in a slowly varying background produced by the oppositely travelling Alfvén waves are just two equivalent descriptions of the same process. We see that the three-wave process gives the correct equation for times comparable to the energy transfer time and the higher order processes can be neglected. This proves that the opposite claim made by GS is invalid. As we have already mentioned, the three-wave weak turbulence theory and the WKB description give the same FP equation also in the case of the Rossby waves [17], indicating that this may be a general property of three-wave systems (e.g., capillary waves, acoustic turbulence, etc.).

It is interesting that there is also a natural transition from the Kolmogorov-type local solutions of the kinetic equations obtained in [13] to the solutions of the FP equation corresponding to nonlocal turbulence. Indeed, the FP equation (11) has a solution $e^+ \propto k_\perp^{-1}$ which corresponds to a constant flux $\hat{D}\nabla_k e^+_{(0)}$. This solution implies very strong asymmetry between e^+ and e^- because the FP equation was derived under the nonlocality assumption where e^- has a much larger scale than e^+ or, in other words, it decreases much faster for large k. On the other hand, constant flux *local* solutions were obtained in [13], $e^+ \propto k_\perp^{n_+}$ and $e^- \propto k_\perp^{n_-}$ with $n_+ + n_- = -4$. It was shown that these solutions are local only if both n_+ and n_- lie within the range from -1 to -3. One can see that at the boundary of locality $n_+ = -1$ there is a continuous transition to the nonlocal solution because the latter has the same spectral exponent. By symmetry, the same is true when n_- reaches -1. Note that k^{-1} seems to be a generic constant flux solution for nonlocal turbulence and can be obtained from a dimensional argument, as shown in [16].

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