MA5N0 Connections, Curvature and Characteristic Classes

Course Organiser: D. Mond Term: 2 24 CATS

Commitment Reading Course

Prerequisites A first course on Manifolds, such as MA455 Manifolds. You will need some familiarity with manifolds and differential forms.

Contents

1. Basic properties of de Rham cohomology and compactly supported de Rham cohomology. Short and long exact sequences.

2. The Mayer-Vietoris sequence: calculation of the de Rham cohomology of spheres.

3. Poincaré duality, and the link between the wedge (cup) product in de Rham cohomology and intersection in singular homology.

4. Applications of de Rham cohomology in degree theory and intersection theory.

5. Morse theory and the Poincaré-Hopf theorem.

6. The cohomology ring of $\mathbb{CP}^n$; complex and Kähler structure on $\mathbb{CP}^n$.

7. Vector bundles and their spaces of sections; operations on vector bundles.

8. Connections on vector bundles and their curvature.

9. Characteristic classes of complex vector bundles, in terms of curvature.

Aims: To understand the role of differential forms in algebraic topology, and to use them to prove a number of important theorems about manifolds, to develop a grounding in Morse theory, to learn the basics of vector bundles and their spaces of sections, to introduce the theory of characteristic cohomology classes canonically associated with vector bundles.

Objectives: Students will learn how integration of differential forms over manifolds provides the key to modern developments in differential topology and geometry.

Assessment Three-hour final exam.

Text: *Printed lectures notes* (of mine) covering the first 60% of the course are available from the Mathematics Institute. The remainder will consist of
1. Chapter 5 of S. Morita, Geometry of Differential Forms, American Math. Soc. Translations of Mathematical Monographs Vol 201, 2000. The main library has a copy (shelf mark QA 381 M6)
2. Chapters 15-18 of *From Calculus to Cohomology*, by Ib Madsen and Jorgen Tornehave, Cambridge University Press, 1997.

The two texts are complementary; Morita gives a much briefer and cleaner treatment, avoiding technical detail. He gives a more motivated account of the curvature of a connection. He should be read first.
Highly recommended for an additional viewpoint: *Differential Forms in Algebraic Topology*, by Bott and Tu, Springer Graduate Texts in Mathematics. This is a modern classic, worth dipping into at various points. If you work with manifolds you will use this book over and over again. But it is rather discursive.