CHAPTERS ON ALGEBRAIC SURFACES

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Foreword

This is a first graduate course in algebraic geometry. It aims to give the student a lift up into the subject at the research level, with lots of interesting topics taken from the classification of surfaces, and a human-oriented discussion of some of the technical foundations, but with no pretence at an exhaustive treatment. I hope that graduate students can use some of these chapters as a reader through the subject, maybe in parallel with a conventional textbook. The early chapters introduce topics that are useful throughout projective and algebraic geometry, make little demands, and lead to fun calculations. The intermediate chapters introduce elements of the technical language gradually, whereas the later chapters get into the substance of the classification of surfaces.

I have given several sets of lectures on algebraic geometry at the graduate or advanced undergraduate level over the last few years. The 1993 Park City summer school, with the challenge of taking a numerous audience through from a basic level up to the sky, was perhaps the most fun. I have thus accumulated a considerable body of notes, without ever having time to organise the material properly in book form. The format of this series allows me to present lecture notes, which are at the same time (somewhat preliminary) extracts of chapters of a more ambitious book on surfaces. A number of the chapters here are based on the notes and examples sheets handed out in class at Park City, with only minor editing. I hope that the reader will excuse their many obvious shortcomings. One of these is the absence of any pictures, a consequence of the way I have put together the notes as computer files; I would not dream of lecturing any part of this material without drawing scribbles every minute or two, and you must make up your own figures if you want the argument to reach deep down to the parts of your intellect that words cannot reach. Some of the chapters have a serious structural fault, namely the tendency to break off into an incoherent list of further topics just as they seem to be getting somewhere. Chapter 4 on singularities should of course be broken up into an introductory discussion of Du Val singularities and the various games we want to play with them, backed up by numerous exercises and worked examples, and a separate chapter giving a serious treatment of resolutions of surface singularities, with adjunction and cohomology; similar remarks apply to Chapter 3 on K3s. I would be very grateful if you would notify me of other possible improvements, and buy my subsequent masterpiece.

Thanks. Some of the material of this book has been lectured at graduate courses at Univ. of Warwick, East China Normal University, Shanghai, Univ. of Tokyo, Univ. of Utah, as well as the Park City summer school. In addition to debts of

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Status of this draft. This is the final preprint of lectures notes to appear in Park City/IAS Mathematics Series, Vol. 3, 1993, pp. 5–159. It incorporates about 100 minor corrections sent in by listeners to the Duke alg-geom server.

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