MA3A6 WEEK 9 ASSIGNMENT : DUE MONDAY 4PM WEEK 9

BILL HART

1. Compute an aribtrary $\mathbb{Q}$-basis for $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$ consisting of algebraic integers and compute the discriminant of that basis. Use this to bound the discriminant of K. Write out a finite list of possible values that the discriminant could be.

2. Now use the algorithm demonstrated in class to determine the discriminant of K. Check your answer with Pari.

3. Let $\mathcal{P} = (2, \sqrt{-5})$ and $\mathcal{Q} = (2, 1 + \sqrt{-5})$ be ideals in the ring $\mathbb{Z}[\sqrt{-5}]$. Compute $\mathcal{P}\mathcal{Q}$ and $\mathcal{P} + \mathcal{Q}$. (Give both the sum and product ideals in terms of one or two generators.)

4. Compute the number of cosets of the ideal $\mathcal{P} = (2, 1 + \sqrt{-5})$ in $R = \mathbb{Z}[\sqrt{-5}]$, i.e. compute the order of R/P and show that $\mathcal{P}$ is a maximal ideal of R. Is it prime?

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