MA3A6 WEEK 6 ASSIGNMENT : DUE MONDAY 4PM WEEK 6

BILL HART

1. Recall that a minimum polynomial of an algebraic number α is the monic polynomial of least degree, with rational coefficients of which α is a root.

Recall also that an algebraic integer is an algebraic number α which is a root of a monic polynomial with rational integer coefficients.

Note that the definition of an algebraic integer does not say that the polynomial is the minimum polynomial of α. Prove that if we take the monic polynomial of least degree with rational integer coefficients that it is indeed the minimum polynomial of α.

Be careful, this is not trivial. Hint: Gauss' Lemma (see textbook).

2. How many units does $\mathbb{Q}(\sqrt{d})$ have if d is a negative fundamental discriminant? (Hint: you will find that there is a general rule with a few exceptions which you can enumerate separately.)

3. Factorise $14 + 12i$ into irreducibles in $\mathbb{Z}[i]$. How do we know there is only one such factorisation upto order of factors and multiplication by units?

4. Prove that $\mathbb{Q}(\sqrt{-2})$ has unique factorisation (in its ring of integers).

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