MA3A6 WEEK 4 ASSIGNMENT : DUE MONDAY 4PM WEEK 4

BILL HART

1. Find a single generator for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$. What degree is the resulting number field?

2. Determine if $\mathbb{Q}(\alpha)$ is galois if α is a root of $f(x) = x^3 - 3x^2 + 2x + 1$.

3. Let α be a root of $f(x) = x^3 - 3x^2 + 2x + 1$. Write each of the following in the form $a_1\alpha^2 + a_2\alpha + a_3$, for $a_i \in \mathbb{Q}$

(i) $\frac{1}{\alpha - 1}$

(ii) $\frac{\alpha^2 - 1}{\alpha}$

(iii) $\alpha^4 + \alpha^2 + 1$.

4. Prove that if $K = \mathbb{Q}(\alpha)$ is a degree n number field then for any $\beta \in K$, the degree of β divides n.

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