MA3A6 "WEEK 11" ASSIGNMENT : NOT ASSESSED

BILL HART

1. Find a group of units of finite index in the full unit group of $\mathbb{Q}(\sqrt{2})$. Write the units of your group in terms of a set of generators for the group and show that all the units represented in this way are actually distinct. Note that you can use Dirichlet's unit theorem for the former, but not the latter.

For those who would like a challenge. Try to find a set of fundamental units for $\mathbb{Q}(\sqrt{2})$. It is fairly easy to find a set of fundamental units, but not so easy to prove it.

By the way, if you do $k = \text{bnfinit}(x^2 - 2)$ and $k.fu$ in Pari, it will compute fundamental units for you, giving the result as a set of elements of the field, each expressed as a polynomial modulo the minimum polynomial of the generator of the field.

2. Use Minkowski's bound to compute the class number (order of the class group) of $\mathbb{Q}(\sqrt{10})$. Recall that Minkowski's result only gives you a bound on the class number, you have to actually find all ideals with norm below that bound to actually determine the class number.

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