MA377 EXAMPLE SHEET III

Q. 1. *Let $\Gamma(n)$ be the directed graph with vertices the set $\{1, 2, \ldots n\}$ and an arrow $i \to i + 1$ for $1 \leq i \leq n - 1$. Let $A_P(n)$ be the path algebra of $\Gamma(n)$. Let $A_T(n)$ be the subspace of $n \times n$ matrices consisting of the matrices such that every entry above the diagonal is zero. Show that $A_T(n)$ is a subalgebra of the algebra of $n \times n$ matrices. Show that $A_P(n)$ and $A_T(n)$ are isomorphic.*

Q. 2. *Let $\Gamma(n)$ be the directed graph with one vertex and n arrows. Let $A_P(n)$ be the path algebra of $\Gamma(n)$. Let $A_G(n)$ be the free algebra on a set with n elements. Show that $A_P(n)$ and $A_G(n)$ are isomorphic.*

Q. 3. *Let X be an infinite set and K a field. Let A be the endomorphism algebra of the vector space with basis X. Show that if we think of an element of A as a matrix then there are only finitely many non-zero entries in each column. Show that the linear operators of finite rank are a proper ideal.*

Q. 4. *Show that if R is left Artinian and does not have zero divisors then every non-zero element has an inverse.*

In different language: Every Artinian domain is a division ring.

Hint: Let x be a non-zero element and consider the left ideals generated by the sequence $x, x^2, x^3, \ldots$.

Q. 5. *Prove that a module is noetherian if and only if every submodule is finitely generated.*