

Convective instability in inhomogeneous media: Impulse response in the subcritical cylinder wake

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(Received 20 May 2010; accepted 17 November 2010; published online 11 January 2011)

We study experimentally the impulse response of a cylinder wake below the critical Reynolds number of the Bénard–von Kármán instability. In this subcritical regime, a localized inhomogeneous region of convective instability exists which causes initial perturbations to be transiently amplified. The aim of this work is to quantify the evolution resulting from this convective instability using two-dimensional particle image velocimetry in a hydrodynamic tunnel experiment. The velocity fields allow us to describe the evolution of wave packets in terms of two control parameters: the Reynolds number and the magnitude of the imposed perturbation. The temporal evolution of energy exhibits a transient algebraic growth at short times followed by an exponential decay.

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I. INTRODUCTION

Flow past a circular cylinder is a classic prototype for studying hydrodynamic instabilities and bifurcations in separated flows.^{1–7} Moreover, the academic case of a two-dimensional cylinder wake can be used as a basic model for many real situations, including the flow behind support cables or around an airfoil. When the Reynolds number $Re = U_0 D / \nu$ (where U_0 is the free-stream velocity, D is the cylinder diameter, and ν is the kinematic viscosity) reaches a particular critical value ($Re_c \approx 47$ for an infinitely long cylinder^{1,2}), a sustained periodic shedding of opposite-signed vortices gives rise to the well-known Bénard–von Kármán vortex street.

The transition to sustained oscillations can be described locally, via wake profiles at different spatial stations, or globally, viewing the two-dimensional (2D) wake as whole. To elaborate further, we first recall the standard distinction between convective and absolute instabilities in parallel flows illustrated in Figs. 1(a) and 1(b). An instability is convective if a perturbation grows but is simultaneously advected with the flow such that the disturbance decays at any fixed point, as in Fig. 1(a), while it is absolute if the perturbation grows at a fixed spatial position, as in Fig. 1(b).^{8–10} A useful way to distinguish convective from absolute instability is in terms of propagating edge or front velocities: assuming a positive leading-edge velocity V^+ , as is the case here, convective instability corresponds to a positive trailing-edge velocity V^- , while absolute instability corresponds to a negative trailing-edge velocity.^{9–13}

Local analysis of wake profiles, i.e., a parallel flow approximation, gives a picture of the transition to sustained oscillations as follows.^{4,13,14} Above $Re \approx 5$ there is a local region of convectively unstable flow in the wake. Above $Re \approx 25$ there is additionally a pocket of absolutely unstable

flow within the region of convective instability. Once the pocket of absolutely unstable flow becomes sufficiently large, the flow becomes globally unstable. Globally, however, one does not observe the onset of a locally absolutely unstable region. Instead, one finds that below Re_c the wake responds to perturbations, but only transiently, as perturbations are advected through the system. This is illustrated in Fig. 1(c). Above Re_c , as in Fig. 1(d), perturbations grow and lead to a synchronized wake in the formation of what is called a global mode.^{5,15} Hence, even though convective and absolute instabilities are strictly defined for parallel flows and streamwise periodic flows,^{9,10,16} inhomogeneous flows, such as flow past a cylinder, may exhibit similar characteristics. The transient response in the subcritical regime has the hallmarks of convective instability while the global instability above Re_c has the hallmarks of absolute instability.

Despite the large body of work on the cylinder wake, only a few experimental studies have examined the wake's subcritical behavior¹⁷ and none have *quantitatively* characterized the transient dynamics in this regime. The goal of the present paper is therefore to analyze quantitatively the subcritical regime using a well-controlled experiment.

After describing the experimental setup, we focus first on the evolution of amplified wave packets and on obtaining the leading-edge, trailing-edge, and group velocities in an experimental setting. The decrease of trailing-edge velocity toward zero when approaching the global instability threshold confirms the transition to an absolute instability. The subcritical behavior is further characterized in terms of the evolution of the maximum amplitude of the wave packet, and its space-time position, as a function of the strength of the perturbation and the distance to the Bénard–von Kármán instability threshold. Finally, to quantify the transient growth phenomenon due to the inhomogeneity of the media, we analyze the temporal evolution of the energy of the perturbation.

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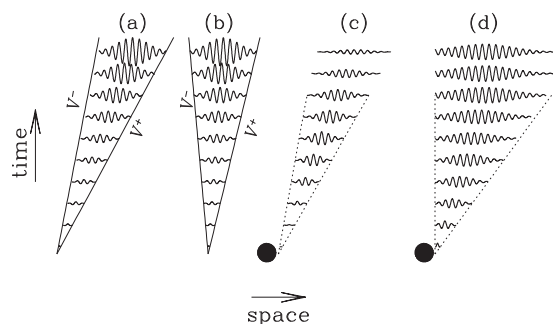


FIG. 1. Sketch of the space-time response of flows to infinitesimal perturbations. (a) and (b) correspond to local analysis, i.e., parallel flows, illustrating the distinction between (a) convective and (b) absolute instabilities. The leading-edge velocity V^+ is positive in both cases. For convective instability, the trailing-edge velocity V^- is also positive, while for absolute instability it is negative. (c) and (d) correspond to the global wake illustrating the distinction between the (c) subcritical, $Re < Re_c$, and (d) supercritical, $Re > Re_c$, cases. In the subcritical case, the perturbation reaches a maximum and subsequently decays. In the supercritical case the perturbation continues to grow until it saturates nonlinearly. The edge velocities can be obtained over finite times only in experiment (see Sec. III).

II. EXPERIMENTAL SETUP

A cylinder of diameter $D=5$ mm is placed in a hydrodynamic tunnel of section $100\text{ mm} \times 100\text{ mm}$ (see Fig. 2) with a nearly plug flow in the test section. The boundary layer width in the tunnel walls is of ~ 10 mm in the region of interest. The cylinder span is 98 mm which covers practically the whole height of the tunnel. We use a Cartesian coordinate system, placed in the cylinder center, with the x -axis pointing in the flow direction and the z -axis running along the cylinder centerline. The cylinder can be put into rotation to provide impulse perturbations to the flow. Measurements are taken in the horizontal midplane of the channel.

We define and work with the reduced Reynolds number $\epsilon = (Re - Re_c) / Re_c$, corresponding to the distance from the global instability threshold. The critical Reynolds number measured in this experiment is $Re_c \approx 64$, which is larger than the ideal 2D case, primarily due to confinement and blockage effects. We adjust the Reynolds number in the experiment by controlling the flow rate in the tunnel. In the results that follow, ϵ varies from -0.30 to -0.04 . For this range of Reynolds numbers, the flow remains 2D over nearly the entire cylinder span.

Impulse perturbations consist of applying very short rotary motions to the cylinder. Rotation is controlled by a programmable microstepping electronic module which gives a resolution of $1/256$ per full step, allowing for a precise control of the cylinder motion. In the present work, we fix the nondimensional time interval over which the cylinder is rotated, and use the speed of rotation as the amplitude of the perturbation. More specifically, the perturbation amplitude is defined from the tangential speed of rotation at the cylinder surface U_p (see Fig. 2). We consider three perturbation amplitudes given by three values of the nondimensional rotation speed: $U_p/U_0 = 75, 100, \text{ and } 125$, which we refer to as small, medium, and strong, respectively. U_0 is the measured velocity of the flat profile. The small-amplitude perturbation

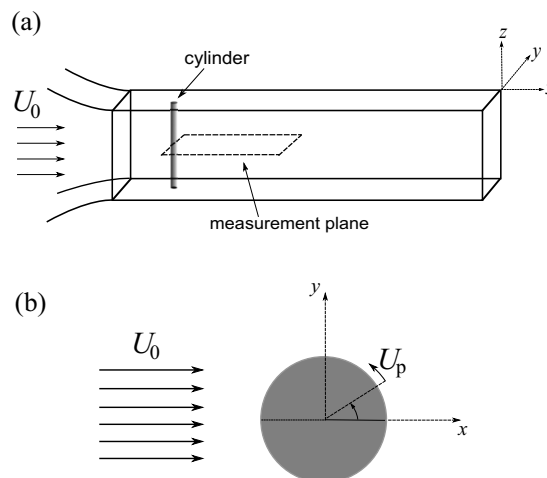


FIG. 2. (a) Schematic diagram of the experimental setup in the hydrodynamic tunnel. (b) Sketch showing a top view of the cylinder during an impulsive rotation of intensity U_p .

is the smallest perturbation that produced an observable response in the wake. For all applied perturbations, the cylinder is rotated for a fix dimensionless duration given by $\Delta t / T_{adv} \approx 0.2$, where Δt is the dimensional duration of the perturbation and $T_{adv} = D / U_0$. Note that since U_0 varies with Reynolds number, both the dimensional tangential speed U_p and the dimensional duration Δt vary with Reynolds number.

The wave packet generated by the perturbation can be observed qualitatively by visualizing streaklines, as in the work of Ref. 17. Figure 3 shows this type of visualization compared with snapshots of the cross-stream component of the velocity field obtained by 2D particle image velocimetry (PIV) in the horizontal midplane. This highlights a significant difference between the present experiment and previous studies of the subcritical wake response. Velocity field measurements obtained by PIV permit one to study directly the spatial structure of velocity perturbations as they evolve. The velocity perturbations exhibit a well-defined maximum in the wake. These are impossible to determine from streakline records since the deformation amplitude of injected dye never decays downstream, an artifact caused by mass conservation of the dyed fluid. Therefore, streakline deformation does not give information about the amplitude of the velocity fluctuations U_y , and thus about the spatial evolution of the impulse response. PIV acquisition and postprocessing have been performed using a LaVision system with an ImagerPro 1600×1200 charge-coupled device camera with a 12-bit dynamic range capable of recording double-frame pairs of images up to 15 Hz and a two rod Nd:yttrium aluminum garnet (15 mJ) pulsed laser. The time lapse between two frames is set to 20 ms. Finally, additional postprocessing and analysis have been carried out with MATLAB and the PIVMAT TOOLBOX.

III. EVOLUTION OF THE WAVE PACKET

The convective nature of the impulsively perturbed cylinder wake is clearly illustrated in the experimental data shown in Figs. 4(a) and 4(b). In Fig. 4(a), cross-stream velocity U_y profiles, measured at successive times along the

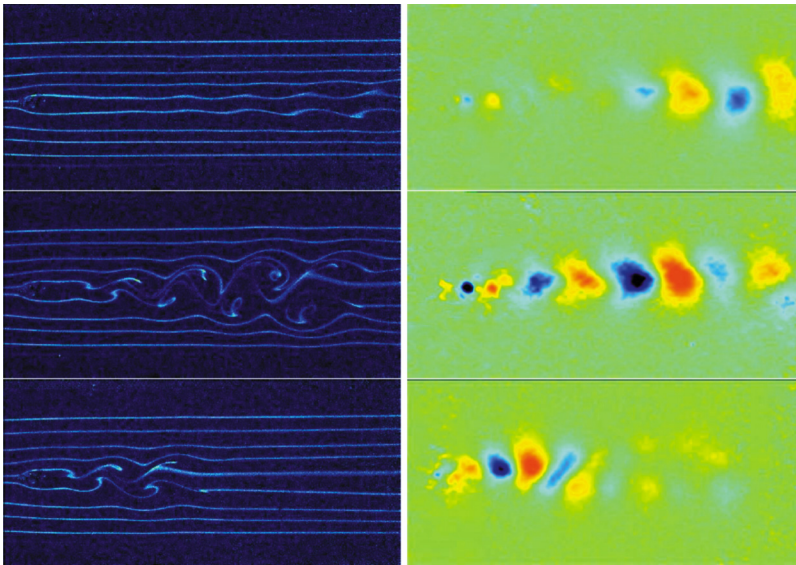


FIG. 3. (Color online) Visualization of the impulse response at three successive time instants (from bottom to top) in the cylinder wake. Left: streaklines obtained from fluorescein dye visualization. Right: instantaneous cross-stream velocity fields obtained from PIV measurements.

wake symmetry axis $y=0$, are stacked up to form a spatio-temporal diagram. The cross-stream velocity on the symmetry axis is the ideal quantity to use to investigate the perturbed flow field because it is everywhere zero for the unperturbed flow. Figure 4(b) is similar to Fig. 4(a) except that the envelope [obtained through a Hilbert transform of U_y as illustrated in Fig. 4(c)] is plotted.

We measure a number of velocities associated with the space-time evolution of the wave packets. From the Hilbert transform of the cross-stream velocity we are able to extract, at each time instant, the envelope of the wave packet as illustrated in Fig. 4(c). Procedurally, we define the leading and trailing positions as the inflection points of hyperbolic tangent fits to the relevant parts of the envelope at each time.

Such fits, together with the associated inflection points, are included in Fig. 4(c). Then we obtain the leading-edge velocity V^+ and the trailing-edge velocity V^- from the collection of fronts extracted from the envelopes at short times ($\leq 50T_{adv}$). Likewise, we obtain the envelope maximum at each time, and from these data, we define the group velocity V_g to be the velocity of envelope maximum, as this gives the speed of the packet as a whole. These velocities are the inverse of the slopes shown in the spatio-temporal diagram of Fig. 4(b). Finally, we define the phase velocity V_p as the translation speed of the vortices in the evolving wave packet as seen in Fig. 4(a). Note that all vortices move at essentially identical speed. Moreover, the leading-edge velocity V^+ , which is effectively determined by the velocity of the first

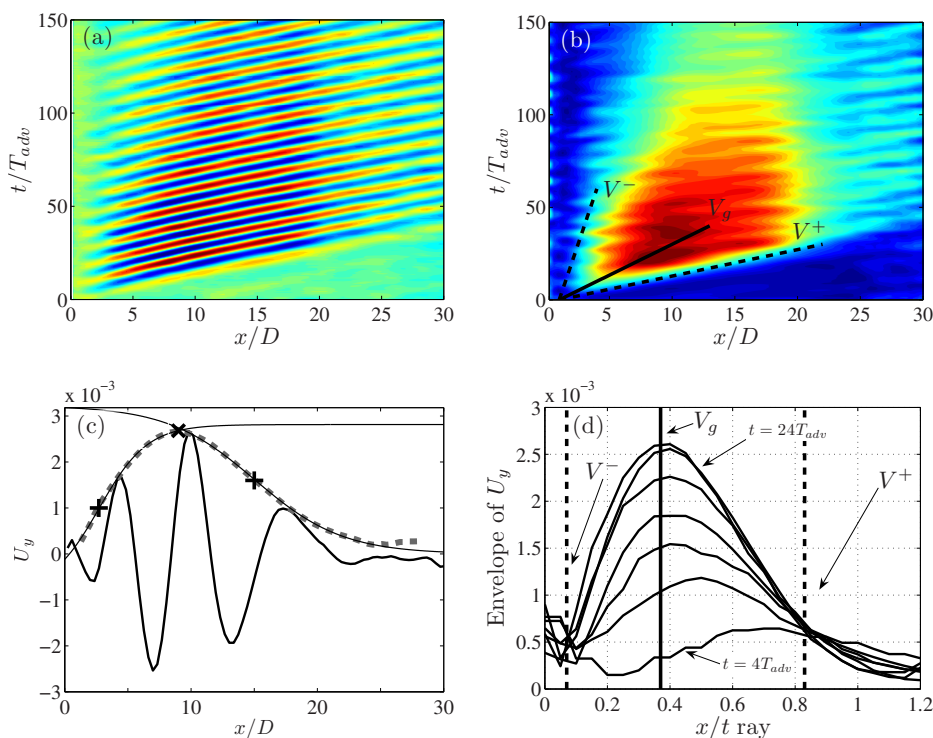


FIG. 4. (Color online) Evolution of the wave packet seen in spatio-temporal diagrams for $\epsilon=-0.137$. (a) Cross-stream velocity $U_y(x,t)$, (b) envelope of $U_y(x,t)$, and (c) $U_y(x)$ and its envelope (bold dashed curve) at a fixed time ($t=24T_{adv}$). Also shown (thin curves) are tanh fits to the relevant part of the envelope. The inflection points and envelope maximum are indicated. (d) Envelope of U_y sampled along various rays as a function of ray velocity. The edge and group velocities, obtained from the inflection points and envelope maxima, are indicated with vertical lines.

advected vortex released from the cylinder, is the same as the phase velocity to within experimental precision: $V^+ \simeq V_p$.

We show now that the edge velocities obtained procedurally through inflection points of the envelope correspond very closely to the edge velocities defined in convectively unstable systems. (See in particular Ref. 13.) In this case one defines V^- and V^+ to be the x/t rays that separate regions of growth from regions of decay in the space-time diagram. The growth rate along rays defining the edges is zero. For the experimental data, the growth rate along rays is shown in Fig. 4(d), where the envelope of U_y , sampled along different x/t rays, is plotted against ray velocity. The edge velocities V^+ and V^- obtained from inflection points are indicated by vertical dashed lines in Fig. 4(d). It can be seen that these are in excellent agreement with the zero growth-rate rays. Moreover, the group velocity V_g (vertical solid line) also agrees very well with the maximum of the envelope in the ray representation.

Thus it can be seen that the velocities are all quite well defined experimentally, at least up to the time at which the perturbation reaches its maximum, even though the flow is spatially inhomogeneous and the wave packet lives only a finite time. As expected, none of the velocities (including V_p) are strictly constant over this space-time region, but they are very nearly so outside of the near-wake region. We have not attempted to extract their detailed variation in the present study.

We now consider the behavior of the different velocities as $|\epsilon| \rightarrow 0$ (i.e., when $\text{Re} \rightarrow \text{Re}_c$). In Fig. 5(a) we can see that the phase velocity V_p , and equivalently the leading-edge velocity V^+ , normalized by the inflow speed, exhibit a slight decrease with increasing Reynolds number. Generally, the value of V_p/U_0 in the subcritical regime ($\epsilon < 0$) compares well with the value of 0.88 reported in the literature¹⁸ for the supercritical ($\epsilon > 0$) regime. The group and trailing-edge velocities are summarized in Fig. 5(b). Both velocities decrease with increasing Reynolds number as one would expect in approaching the global wake instability. To within experimental accuracy, V^- approaches zero as $\epsilon \rightarrow 0$.

From the spatiotemporal diagrams we are also able to pinpoint the position in space and time (X_{\max}, T_{\max}) at which the subcritical response reaches maximum amplitude. See Fig. 6(a). Again we use the cross-stream velocity component U_y . We let A_{\max} denote the maximum of the response, so that $A_{\max} = U_y(X_{\max}, T_{\max})$.

Figures 6(b)–6(d) show the dependence of the maximum on reduced Reynolds number. It is not surprising that A_{\max} grows when the global instability threshold is approached ($|\epsilon| \rightarrow 0$), as can be seen in Fig. 6(b), because the susceptibility of the flow increases near the onset. On the contrary, the behavior of X_{\max} (which diminishes when $|\epsilon| \rightarrow 0$, see Fig. 6(c), deserves a further comment, since it seems to be opposite to the case of other instabilities in the subcritical regime where the characteristic length scale increases when $|\epsilon| \rightarrow 0$ (e.g., the penetration length in pretransitional Rayleigh–Bénard convection¹⁹). The fact that the maximum of the instability moves closer to the cylinder (i.e., that X_{\max} diminishes) when approaching the threshold while the time at which this occurs remains constant [on a nondimensional

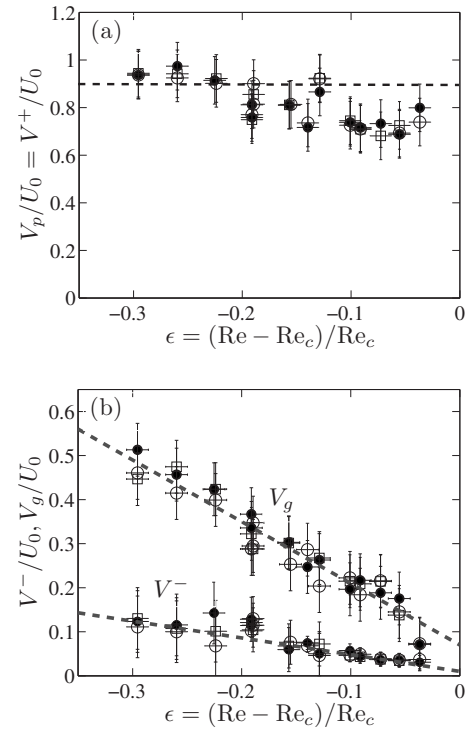


FIG. 5. (a) Phase velocity V_p (which is also V^+) and (b) group velocity V_g and trailing-edge velocity V^- as a function of ϵ . The dashed line in (a) corresponds to the value of V_p/U_0 for the Bénard–von Kármán wake ($\text{Re} > \text{Re}_c$). The dashed lines in (b) are linear fits to the V_g and V^- data. Experimental points are labeled as ●: small perturbation, ○: medium perturbation, and □: strong perturbation. Horizontal error bars indicate the uncertainty in the Reynolds number measurement.

scale normalized by the advective time scale T_{adv} , Fig. 6(d)] is actually consistent with V_g decreasing as the convectively unstable system tends to the absolute instability threshold. The difference with the case of Rayleigh–Bénard convection comes from the effect of the mean flow advection, which modifies the physical meaning of the penetration length scale.

For a given value of ϵ , an increase in the strength of the perturbation produces a response in which the maximum amplitude A_{\max} is slightly larger and occurs at a position closer to the cylinder (smaller X_{\max}). Approaching the threshold reduces the effect of changing the perturbation strength.

IV. TRANSIENT GROWTH

As noted in Sec. I, an important feature of the cylinder wake is the inhomogeneous nature of the flow. Hence, even though the subcritical response just presented has much of the general character of convective instability, perturbations do not grow indefinitely (even linearly) as they would for a homogeneous convectively unstable system. Rather, a localized initial perturbation grows at first, due to local flow features near the cylinder, but is simultaneously advected downstream into a region of stability where the perturbation decays. Hence in the absence of any inflow noise the impulsive response is only transient. Such behavior is known in inhomogeneous flows.^{5,20–27} It should be noted, however, that inflow or other noise may modify the picture in that the

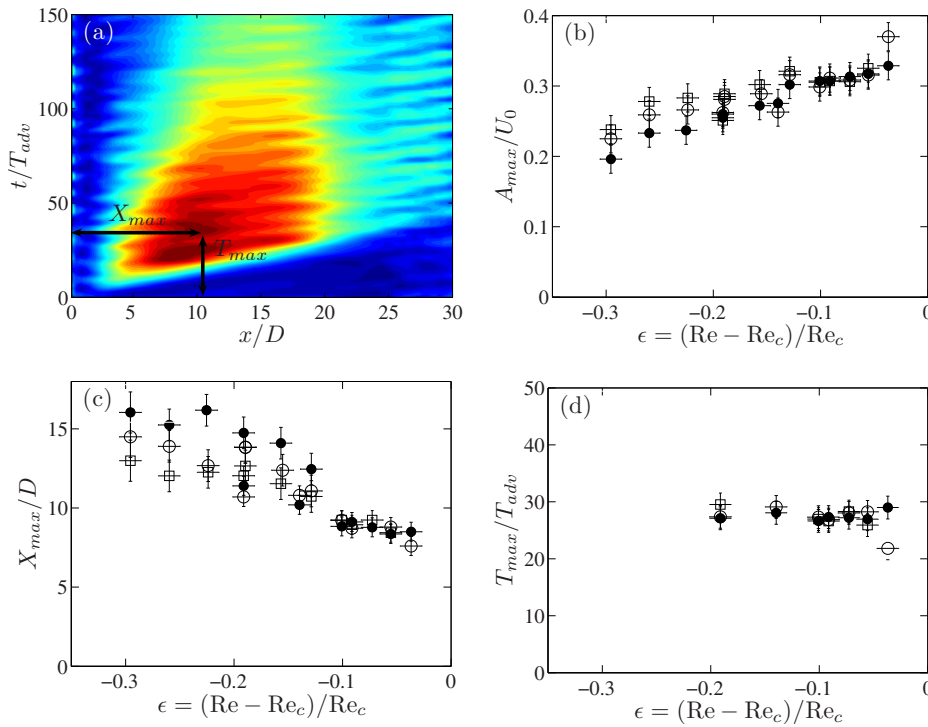


FIG. 6. (Color online) (a) Definition of the maximum of the perturbation on the spatio-temporal diagram. (b) Evolution of the maximum of the perturbation A_{max} , (c) X_{max} , and (d) T_{max} as a function of the reduced Reynolds number ϵ . A_{max} , X_{max} , and T_{max} are rendered nondimensionalized, respectively, by the free-stream velocity U_0 , the cylinder diameter D , and the advective time scale $T_{adv} = D/U_0$. Experimental points are labeled as \bullet : low perturbation, \circ : medium perturbation, and \square : strong perturbation.

localized region of instability acts as an amplifier and sustained dynamics may arise in some cases even in the subcritical regime.^{11,24,27–29}

An increasingly common approach to quantifying the transient response of flows is in terms of their transient energy growth.³⁰ Such an analysis provides a global measure of the response dynamics. Here we undertake such an analysis of experimental data.

We define the perturbation energy from our measurements as follows

$$E(t) = \int_{\Delta} \int_{\Delta} (u_x^2 + u_y^2) dx dy,$$

where

$$u_x = U_x - U_{x \text{ base}}; \quad u_y = U_y - U_{y \text{ base}},$$

where the base flow ($U_{x \text{ base}}, U_{y \text{ base}}$) is the measured steady flow before any perturbation.

Experimentally, the total energy of the perturbation can only be measured while the packet is contained in the observation window. Hence, the energy calculated from the velocimetry data does not include the contribution from vortices that have been advected out of the measurement area. In order to quantify this effect we compare the energy calculated using two different streamwise sizes for the integration area Δ : the total energy E_{tot} , where the whole measurement window is used, and the energy of the first half of the wave packet E_{half} , where the downstream boundary of the integration area Δ is given by the time-dependent position of the maximum perturbation amplitude $\tilde{X}_{max}(t)$; see Fig. 7(a). We note that $\tilde{X}_{max}(t)$ is time-dependent and should not be confused with X_{max} shown in Fig. 6(c). The quantities are related

via: $X_{max} = \max_t \tilde{X}_{max}(t)$. We find that the ratio of E_{tot}/E_{half} remains approximately constant (≈ 2) in time, which means that the measurement window is sufficiently large to capture the dynamics of the perturbation growth and decay before the effect of the flow structures advected away from the downstream boundary of the measurement window becomes significant.

The time evolution of the energy is shown in Fig. 7(b). The value of E_{tot} , as well as E_{half} and $E_{tot} - E_{half}$, are shown. All energies are all normalized by E_0 , the value of E_{tot} at the first measured instance following the impulse. The perturbation energy initially undergoes growth until $t \approx 50T_{adv}$, at which point the energy decays. This is precisely the transient growth dynamics expected of convective instabilities in inhomogeneous media.

The late time behavior of the energy corresponds to the exponential decay of the least stable normal mode in a stable region, e.g. Refs. 31 and 7. As the system approaches the absolute instability threshold ($\epsilon \rightarrow 0$) this decay becomes slower. This can be seen in Fig. 7(c), where the decay rate is given by the slope of the curves of $\log E_{tot}$ and more quantitatively in Fig. 8(a) where the asymptotic decay rate is plotted as a function of reduced Reynolds number. For comparison, decay rates from linear stability computations⁶ are shown with a solid curve. The agreement is excellent.

The energy growth at short times can be examined using the maximum of the energy E_{max} and the time for which this maximum is reached t_{max} as a function of ϵ . See Fig. 8(b). Consistently with the approach of the absolute instability threshold, E_{max} increases when $|\epsilon| \rightarrow 0$. The time t_{max} where the maximum energy is reached remains approximately constant.

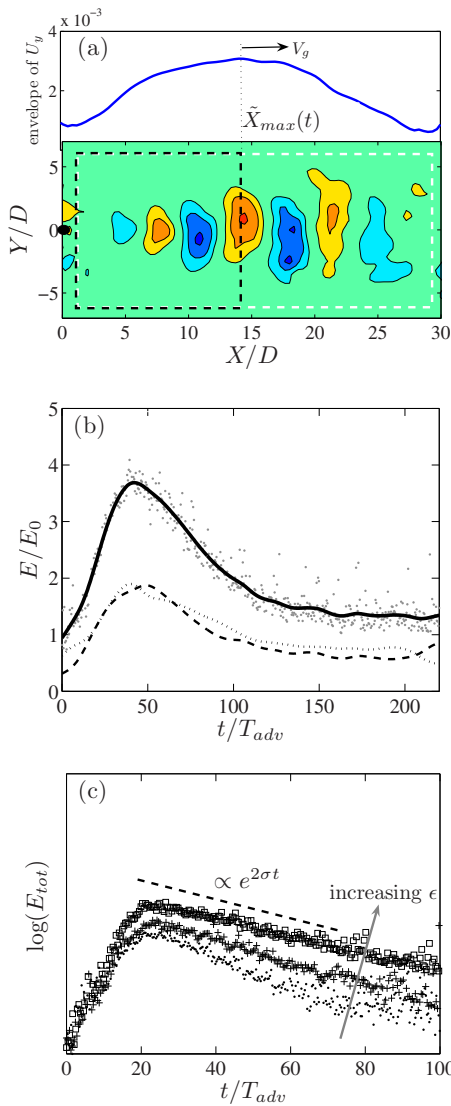


FIG. 7. (Color online) Transient energy growth in experiment. (a) Illustration of the integration area for the energy on the instantaneous velocity field U_y . E_{tot} and E_{half} are calculated over the white and black rectangles, respectively. The downstream boundary of the black rectangle moves at a speed V_g and tracks the maximum of the perturbation envelope as indicated in the upper plot. (b) Energy as a function of time for $\epsilon = -0.14$. The dots are the experimental data of E_{tot} . The solid, dashed, and dotted lines are smooth fits of E_{tot} , E_{half} , and $E_{tot} - E_{half}$, respectively. (c) Time evolution of the energy on logarithmic scale, for three different values of the reduced Reynolds number ϵ : \bullet $\epsilon = -0.19$, $+$ $\epsilon = -0.14$, and \square $\epsilon = -0.1$. The slope of the energy decay on the logarithmic scale gives the decay rate σ .

V. DISCUSSION AND CONCLUSIONS

The convectively unstable wave packets produced by an impulsive perturbation in a subcritical cylinder wake have been studied experimentally. Velocity field measurements obtained by PIV have permitted us to characterize quantitatively the instability wave, shedding light on points that remained not clearly analyzed in the literature. First, probing the perturbation of the velocity field due to the instability shows unambiguously that there is a well-defined maximum of the perturbation amplitude in the wake. This differs significantly from the picture given by the streakline visualizations usually used to illustrate the convective instability,¹⁷ which distorts the observation of the actual growth and de-

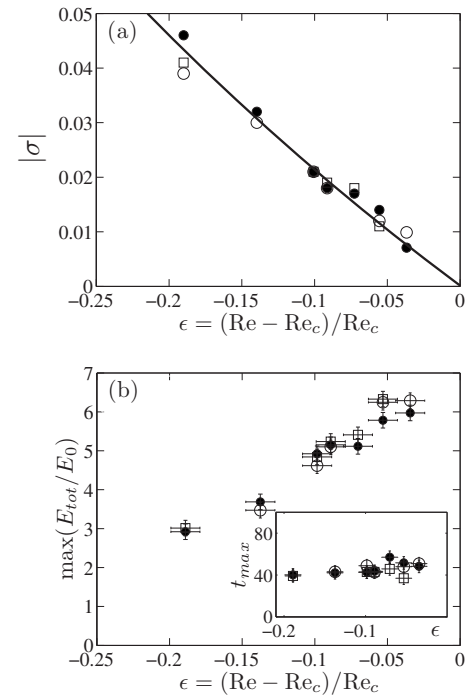


FIG. 8. (a) Asymptotic decay rate of perturbations [as seen in Fig. 7(c)] as a function of reduced Reynolds number ϵ . Experimental points are labeled as \bullet : low perturbation, \circ : medium perturbation, and \square : strong perturbation. The curve shows decay rates from linear stability computations. (b) Maximum transient energy gain and (inset) time at which this maximum occurs as a function of the reduced Reynolds number ϵ . Experimental points are labeled as \bullet : low perturbation, \circ : medium perturbation, and \square : strong perturbation.

cay. The value of this maximum and its position downstream of the cylinder depend on the Reynolds number and, less markedly, on the strength of the perturbation, whereas its position in time remains constant. In addition, the evolution of the wave packet has been characterized with respect to the Reynolds number using the leading and trailing fronts as well as a typical group velocity. The measured velocities are consistent with the transition from a convective to an absolute global instability as the Reynolds number increases toward the Bénard–von Kármán instability threshold (see Fig. 5).

The analysis of the transient energy growth associated with the instability also deserves a final comment. The qualitative features of the temporal evolution of the energy agree with the standard picture of transient growth due to convective instability in inhomogeneous media, i.e., a short-time algebraic growth followed by an exponential decay at late times. However, the measured values of $\max(E_{tot}/E_0)$ are remarkably low (always less than 10) when compared to the values obtained by numerical computations (of order 10^3 , see e.g. Refs. 25 and 27). The main reasons for this discrepancy are most likely the fact that in the experiment one does impose an *optimal* perturbation and also the fact that extracting E_0 , the initial energy of the perturbation, is experimentally quite difficult. This discrepancy raises the question about the pertinence of the energy gain (ubiquitous parameter in transient growth studies) as the most appropriate quantity to use for comparison between theory and experiments.

ACKNOWLEDGMENTS

D.B. gratefully acknowledges support from the Leverhulme Trust and the Royal Society. We acknowledge support from the French Research Agency through Project No. ANR-08-BLAN-0099.

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