Sixth Postgraduate Group Theory Conference

Notes For Participants

Introduction

Welcome to the Sixth Postgraduate Group Theory Conference here in Warwick. We hope that you will find the conference enjoyable and interesting.

1 General arrangements

All seminars will be held in lecture room MS04, on the second floor of the Mathematics Institute. An overhead projector will be available. Tea, coffee and biscuits will be available during breaks in the common room which is on the first floor of the building.

There will be access to the department's computer facilities via a personal password which we will give you on arrival, should you wish to check your email. The department's computers can be persuaded to run LaTeX, magma, and most other common mathematical software, but they are slow Unix based systems, so it would probably be easier to do anything complicated before you come.

You will not have access to rooms in Rootes until around 3pm on the Wednesday, and must check out of Rootes Residences by around 9:30am on Friday 26th. We have made arrangements to store luggage securely at the Mathematics Institute at times when rooms are unavailable.

2 Food and drinks

For the duration of the conference all main meals are provided.

There will be a buffet style dinner in the Institute Common room on the evening of Wednesday 24th April.

This will include: Boned stuffed chicked, various quiches, marinated mushrooms, pasta salad, coleslaw, tomato and green salads, and a selection of fresh fruit and desserts.

Lunch on Thursday will be sandwiches in the common room of the Mathematics Institute. In the evening the conference dinner will be held at the Sutherland Staff Lounge in Rootes Social Building (47 on map).

Menu:

Starter: Chilled Roasted Pineapple with Orange: Slices of roasted pineapple marinated in a fresh orange glaze, dusted with spices, served chilled

Main: Braised Blade of Beef: A braised blade of beef with caramelised button onions and a rich red wine jus

OR Roasted Beef Tomato: A beef tomato filled with ragout of beans, topped with brie, served with a herb butter sauce

Dessert: Baked Chocolate Cheesecake: An individual baked cheesecake, placed on a pool of lemon thyme sauce Coffee with Chocolates

Our Friday lunch will be at EAT at 1pm, the restaurant in Warwick Arts Centre (number 58 on map).

Menu:

Starter: Tomato and Orange Soup

Main: Supreme of Chicken with Pink Peppercorn Sauce

OR Beer Battered Haddock fillet with mushy peas

OR Roast Vegetable Hot Pot

Dessert: Queen of Puddings

The accommodation at Rootes includes breakfast on Thursday and Friday mornings which will be available from Rootes Restaurant, in Rootes Social Building between 7:30am and 9am.

The official conference bar will be "The Bar" in Rootes Social Building, although we may also use the facilities in the students union building which is the next-door building.

3 Guest speakers

The opening and closing talks will be given by our guest speakers, Prof. Rick Thomas (Leicester) and Dr. Brent Everitt (York). These will last for around an hour.

Rick Thomas

Title: Groups and formal languages.

Abstract. One of the classical results in combinatorial group theory is the unsolvability of the word problem for finitely presented groups; this says that there are finite presentations such that there is no algorithm to decide whether or not a word in the generators (and their inverses) represents the identity element of the group defined by the presentation. An alternative way of describing

this situation is as follows: there are finitely presented groups G such that, if we consider the set W of all words representing the identity element of G, then there is no algorithm for determining membership of W. There is also an elegant result of Boone and Higman describing which finitely generated groups have a solvable word problem.

One natural question that arises from this is the following: if we take some restricted model of computation, which groups have a word problem which is decidable within that model? This relates to "formal language theory" where classes of languages are described in terms of (for example) a type of abstract machine that determines membership of such a language. (A "language" in this setting is just a set of words.)

The purpose of this talk is to survey some of what is known in this field. We will not assume any prior knowledge of formal language theory and only limited notions from the theory of groups.

Brent Everitt

Title: Manifolds: It's all done with mirrors

Abstract: Groups that are generated by reflections appear throughout nature. Their abstract manifestations, Coxeter groups, have many fascinating connections with geometry, topology, number theory and mathematical physics. This talk focuses on one particular application of Coxeter groups, namely to the construction of hyperbolic manifolds. These are spaces in which one can locally do hyperbolic geometry. The holy grail (at least in this talk) is to find an answer to the following question: what is the shape of the universe?

4 Provisional Timetable

Student's should last for approximately 20 minutes.

	Wednesday 24th	*Thursday 25th*	*Friday 26th*
9:30		Seminars	Seminars
11:00		*Coffee*	*Coffee*
11:30		Seminars	Seminars
12:00	*Registration*	Seminars	Seminars
12:30		Seminars	Seminars
13:00		*Lunch: Sandwiches*	*Lunch: EAT*
13.50		Seminars	
14:00	Rick Thomas	Seminars	
14:30		Seminars	Brent Everitt
15:00	Coffee	Coffee	
15:30	Seminars	Seminars	
16:30	Seminars	Seminars	
17:00	Seminar	Seminar	
	Buffet Dinner: 6pm	*Dinner: 7pm*	
	Institute Common Room	*Sutherland Suite*	

5 Abstracts

If you are giving a talk but have not yet sent us your title and/or abstract, please do so as soon as possible. We have already received the following titles and abstracts, and hope to have a full list ready for the final information.

David Webdale - An automatic structure on infinite Coxeter groups

I intend to present a brief summary of what infinite Coxeter groups, and automatic structures mean, before moving on to combine the two concepts. A finiteness condition on a subset of the root system of the Coxeter group will provide the key fact for the construction of the automatic structure. I will state the results of some calculations of the size of this subset that I have carried out, and give an example. I will then concentrate on constructing the word acceptor automaton of the automatic structure. I will leave the construction of the multiplier automata alone, but note that is also based on the same finiteness condition.

Matt Owens - Automatic Groups and Generalizations.

An automatic group is a finitely generated group characterized by a finite set of FSA. This talk will examine some basic properties and open problems, before examining generalizations of (and diversions from) the theory.

Mark Stather - Constructing a Chief Series of a Matrix Group

The aim of this talk is to describe an algorithm that uses Aschbacher's Theorem on the Classification of Subgroups of $GL(d, q)$ to construct a chief series of $G < GL(d, q)$. The chief series and chief factors are constructed from kernels and images of maps brought about by the specific Aschbacher Class that G belongs to. Hopefully a Magma implementation of this algorithm will be available for download somewhere on the web by April.

Simon Nickerson - Rational Reconstruction of Representations

For any natural number m, let $\mathbb{Q}_m$ be the set of rationals a/b with b coprime to m, and let θ be the map $\mathbb{Q}_m \to \mathbb{Z}_m$ which sends a/b to $\bar{a}/\bar{b}$, its reduction modulo m. *Rational reconstruction* is the process of finding a 'small' θ-preimage of an element of $\mathbb{Z}_m$. We will outline a well-known technique for rational reconstruction using Euclid's algorithm. Then, using an example, we will show how rational reconstruction can dramatically speed up the decomposition of a group representation into its irreducible constituents.

Catarina Carvalho - One Application of Reidemeister-Schreier type rewriting for semigroups

A Reidemeister-Schreier type method for writing presentations of subsemigroups of semigroups was developed by C. M. Campbell, E.F. Robertson, N. Ruškuc and R.M. Thomas in *Reidemeister-Schreier type rewriting for semigroups* (Semigroup forum vol.51, 1995, 47-62).

We will describe this method and show how it was used to prove that if the Bruck-Reilly extension of a Clifford monoid is finitely presented then the monoid must be finitely generated.

Elizabeth Wharton - Identities, quasi-identities and universal theory

During the 1960s the following problems were presented:

- describe the universal theory of free nilpotent groups of class c;
- describe the universal theory of free groups;
- describe the system of quasi-identities true in free groups.

They are all still open. The first is very hard but some progress has been made in the latter problems.

We define universal theory and quasi-identities and discuss some results in an area related to these problems - descriptions of the quasi-identities and universal sentences true in groups free in certain soluble varieties.

Graham Oliver - Automatic presentations: a different notion of automaticity

The structure of a group may be investigated by considering how complicated the group operation is computationally. A group is said to be computable if its domain and operation can be read by Turing machines. Restricting the Turing machines in the definition of computable groups to finite automata then gives us groups with particularly simple computational structure. These groups are said to have an automatic presentation. This talk will outline the proof of the classification: a finitely generated group has an automatic presentation if and only if it is virtually abelian.

Richard Bayley - Young Tableaux and the K-S-K Correspondence

Young Tableaux are fillings of boxes of diagrams which correspond to partitions of positive integers, which are weakly increasing across rows and strictly increasing down columns. The aim of the talk is to give a brief insight in to the basic combinatorics of Young Tableaux, including the bumping and sliding algorithms. Schensted, C (Longest increasing and decreasing subsequences, Canadian Journal of Mathematics 13 1961) is the classic paper for this topic, and it is here that an application is for Young Tableaux is given in finding increasing and subsequences of a list of integers. We shall concentrate on the Robinson-Schensted-Knuth correspondence which allows us to associate an ordered pair of tableaux of the same shape (P,Q) to an arbitrary two-rowed array which is in fact a bijective correspondence between the set of pairs of tableaux of the same shape and the set of two rowed arrays. We will conclude by showing some interesting applications of the correspondence and using the hook length formula to count tableaux of a certain shape.

Robert Bailey - Permutation Groups, Error-Correcting Codes and Covering Designs

Traditionally, coding theory makes use of *linear codes*, which are vector spaces over finite fields. We replace these with permutation groups, and show that certain group-theoretic properties (e.g. minimum degree, bases) have some meaning in this setting. We shall draw upon sharply k-transitive groups as well as the general linear and affine general linear groups for examples.

Louise Archer - Quotients of Hall Algebras

Given a finitary algebra A, we can construct the Hall algebra $H(A)$, which is a Q-algebra with multiplication given by counting filtrations of A-modules. These are infinite dimensional algebras and are, in general, difficult to describe concretely. In this talk we will look at some quotients of Hall algebras, including some examples which can be fully described in terms of generators and relations.

Sophie Whyte - Constructing images of progenitors

Abstract to follow

Pablo Spiga - Local partitions in finite simple groups

A partition for the elements of prime power order in a finite group G is a family of subgroups with the property that every non-identity element of prime power order lies in exactly one subgroup on the family. The main result of this talk is a classification of the finite simple groups which have such a partition. We also show the connection between this concept and the class of permutation groups where the numbers of fixed points of elements of prime power order are all equal.

Ahmad Alghamdi - Predictions of Conjectures for p-blocks with an Extra Special Defect Group

Let G be a finite group, p be an odd prime number and B a p-block of G with defect group E which is an extra special p-group of order p^3 and exponent p. Consider a maximal (G, B)-subpair (E, b_E). It is well-known that B corresponds to a unique p-block of $N_G((E, b_E))$, say b, with defect group E. Then the Ordinary Weight Conjecture holds for B if, and only if, $k_d(B) = k_d(b)$, for all non-negative integers d, where $k_d(B)$ means the number of irreducible characters of B with defect d.

Sarah Campbell - Geometry and Exactness

We say that a group Γ is exact if the operation of taking the reduced crossed product with Γ preserves exactness of short exact sequences of Γ-C^*-algebras. Establishing whether a group is exact or not has traditionally been done by very analytic methods, using results by Kirchberg and Wassermann. However, recent results by Ozawa, Guentner and Kaminker enable us to use a far more geometric approach. We will introduce one such method, giving an example by Guentner and Kaminker which uses the structure of trees to establish that free groups are exact. We will then briefly discuss how this can be modified to show that groups acting properly and cocompactly on CAT(0) cube complexes are exact.

Jerry Swan - State-space search for pure mathematicians

There are a number of techniques from Computer Science that might profitably be applied to problems in Pure Mathematics. Starting with an explanation of some forms of "blind" search, we go on to discuss subsequent developments, including Genetic Algorithms, which have already been applied to a number of group-theoretical problems with varying degrees of success.

Elizabeth Kimber - Generalized cyclic presentations of finite groups

A necessary condition for a finite group G to be cyclically presented is that it has an element x and an automorphism θ such that the set $\{x\theta^i : i = 1, 2, \ldots, o(\theta)\}$ generates G. However, there are finite groups that satisfy this conditon, but cannot have a cyclic presentation because they do not have deficiency zero. For example, $C_2 \times C_2$ has non-trivial Schur multiplier, so it cannot have a deficiency zero presentation. These groups are said to have a generalized cyclic presentation (GCP). Every finite abelian group has a GCP, so a natural question to ask is when does an abelian group have a GCP on a minimum generating set? Another question is when does $G \times H$ have a GCP if G is abelian and H is non-abelian? Example based partial anwers to both these questions will be given.

Tim Honeywill - A casual stroll through the combinatorics related to Kazhdan-Lusztig theory

I am not really a group theorist but rather a Lie theorist who happens to play around with certain Coxeter groups in my work. I therefore hope to only give a flavour of what I do, rather than worry with the details, aware that I will be describing terminologies and definitions which are possibly new to many in the audience. The Weyl groups and affine Weyl groups that arise in Lie theory are examples of finite and infinite Coxeter groups. In a famous paper by Kazhdan and Lusztig in 1979, equivalence classes, called cells, were defined on these groups. These cells play an important role in the representation theory not only of the Coxeter group and a certain algebra to which it is associated (called the Hecke or affine Hecke algebra) but also of many other objects in Lie theory. Although the deep theory underlying Kazhdan-Lusztig cells can seem very complicated and draws on many different areas of mathematics (in other words, a lot of the theory I do not understand!), amazingly, a lot of combinatorial patterns and machinery can be used to described these cells. The first observation of this, for example, was that the Robinson-Schensted Correspondence (an algorithm which produces pairs of same-shape standard Young tableaux) describes exactly the Kazhdan-Lusztig cells of the symmetric group, the 'simplest' Weyl group arising in Lie theory. Twenty minutes is not long, but I hope to at least describe some of the combinatorial machinery used and some of the patterns observed.

Rachel Abbott - Representations of $L_3(8)$

In my talk I will discuss the representations of $L_3(8)$ over the field of order 8. I shall begin by considering the construction of the field using the relevant Conway Polynomial. The Meat-Axe, discussed in R.A.Parker's paper of 1984, uses the numbers $\{0, 1, 2, 3, 4, 5, 6, 7\}$ to represent the field elements and I will demonstrate this correspondence. I will go on to talk about how the representations can be constructed using tensor products of the natural representation and the Meat-Axe. I will give examples of the matrices used to generate some of the smaller ones. In addition to this I will talk about how representatives for the conjugacy classes of $L_3(8)$ can be found using the natural representation. I will finish with a brief word on how these techniques can be generalised to representations over other fields and how my work will contribute to the web atlas.

John Bradley - Symmetric Presentations

We will discuss the idea of a symmetric presentation and, in particular, a symmetric presentation of the McLaughlin sporadic simple group.

Anton Evseev - Reduced zeta functions of groups

A local zeta function of a nilpotent group is a generating function counting (normal) subgroups. We investigate the properties of a 'reduction modulo 1' of such a zeta function.

Russell Fowler - Introduction to the theory of spherical orbits.

In my talk I will introduce the concept of G acting on X where G be an algebraic group, and X an affine variety. I will pay particular attention to the action of G on its Lie algebra $L(G)$ and also the quotient space G/H, for H a subgroup of G. I will include results of D.I.Panyushev which give surprisingly simple necessary and sufficient conditions for G-orbits on $L(G)$ to be dense. In addition I will discuss some theorems of F.Knop on the finiteness of Borel orbits and the theory of spherical orbits and subgroups.

Attila Maroti - A short proof for a Nakayama conjecture

Recently, Külshammer, Olsson and Robinson generalized the Nakayama conjecture for symmetric groups. In this talk, by a similar but a shorter argument we prove more.

Mark Wildon - Estimating $p(n)$

A partition of a positive integer n is a non-increasing sequence of positive integers $\lambda_1, \lambda_2, \ldots, \lambda_k$ whose sum is n. Let $p(n)$ be the number of partitions of n. By rather sophisticated methods Hardy and Ramanujan proved in 1918 that

$$p(n) \sim \mathrm{e}^{\pi\sqrt{2n/3}} \Big/ 4\sqrt{3}n \quad \text{as } n \to \infty.$$

My talk will give some upper and lower bounds for $p(n)$ that can be obtained by combinatorial methods. I shall also attempt to bring out the connection with the representation theory of the symmetric group.

Alexander Stasinski - Representations of reductive groups over finite rings

The classical work of Deligne and Lusztig uses methods from algebraic geometry to construct (complex) representations of reductive algebraic groups over finite fields. This construction can be generalised to reductive groups over finite rings coming from the ring of integers in a local field, modulo some power of the maximal ideal. Lusztig conjectured that all irreducible representations of these groups are contained in the cohomology of a certain family of varieties. We will give an overview of a proof showing that this conjecture does not hold in general. Moreover, we show how the missing representations in the case under consideration, can be realised by a different kind of variety. This opens up the problem of reformulating Lusztig's conjecture.

Marie Vernon - Finding a basis for Young modules

I will review the definitions of permutation modules and Young modules for the symmetric group, in order to consider the problem of determining a basis for Young modules. I will give some examples to illustrate this, and state some conjectures.

Keith Goda - Solving equations in groups - centraliser dimension in graph products.

Algebraic geometry over groups is a subject of much recent work. I look in particular at finding the centralisers a particular class of groups, called a graph product. The graph product is a generalisation of the notions of free and direct product. Centralisers are the sets of solutions of the equations $gx = xg$. The notion of *centraliser dimension*, to be described, gives us an invariant of the group.

We are also expecting talks from:

Matt Towers, Roshani Nanayakkara