CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

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Observation of Wave Turbulence on a fluid surface

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Zero gravity experiment:

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Wave Turbulence

• Study of the dynamical and statistical properties of a set of dispersive waves with nonlinear interactions

• Occurs at very different scales in various systems : hydrodynamic, astrophysics, optics…

• Laboratory exp. are scarce / studies in 2D and 3D hydrodynamic turbulence !

Goals: Wave Turbulence on a fluid surface

I. Characterize the transfer of energy injected on large scale cascading towards the small structures, through the wave interactions, dissipating the energy at the end of the cascade

$\Rightarrow$ Measurement of the spectrum and the distribution of the wave amplitude fluctuations

II. Know the statistical properties of the fluctuations of the energy flux necessary to bring a dissipative system out-of-equilibrium

$\Rightarrow$ Measurement of the fluctuations of injected power into the fluid

III. Wave turbulence in low gravity environment?

Experimental Setup

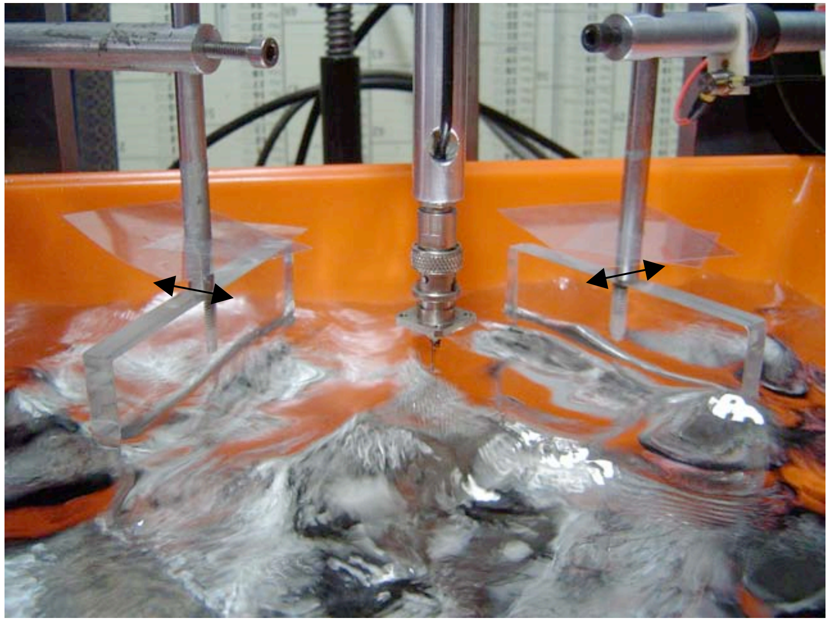
- Rectangular vessel : 20 x 20 cm^2 or 57 x 20 cm^2

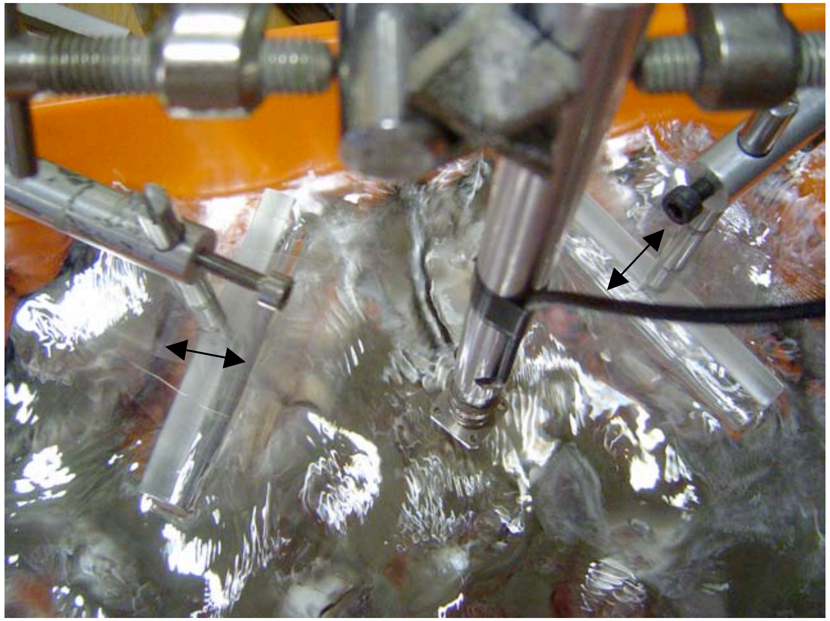
- Fluid: Mercury or Water, depth ~ 2 cm

- Wavemakers driven with low frequency random forcing: 0 - 6 Hz ⇔ $\lambda > 5$ cm

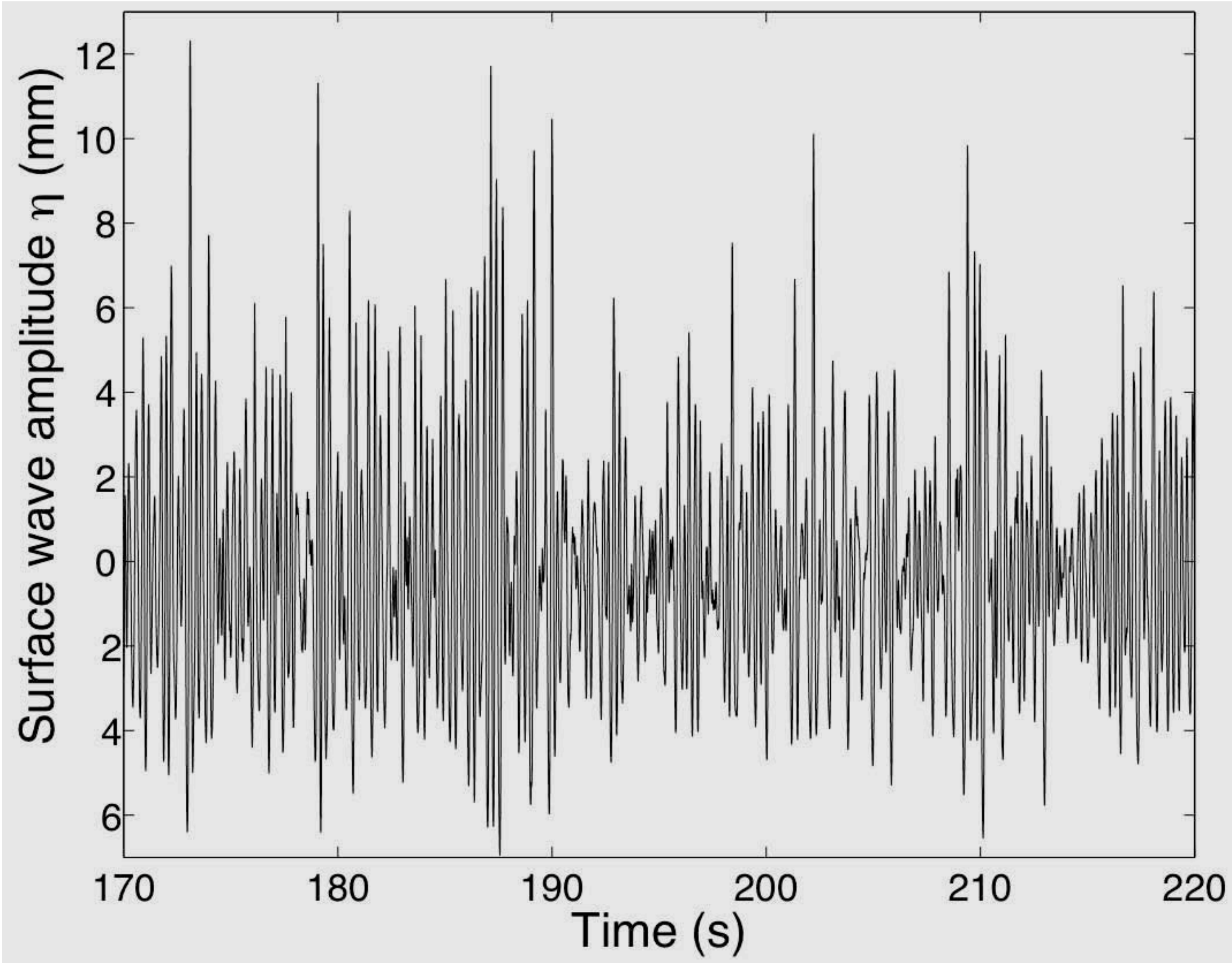
≠ Faraday forcing: Wright et al. PRL (1996) ; Henry et al. EPL (2000); Brazhinikov et al. JETP (2002)
≠ *in situ* experiments on the ocean surface (winds, ocean current) : oceanographers

- Amplitude of the surface waves measured with a capacitive sensor (wire ϕ =0.1 mm)



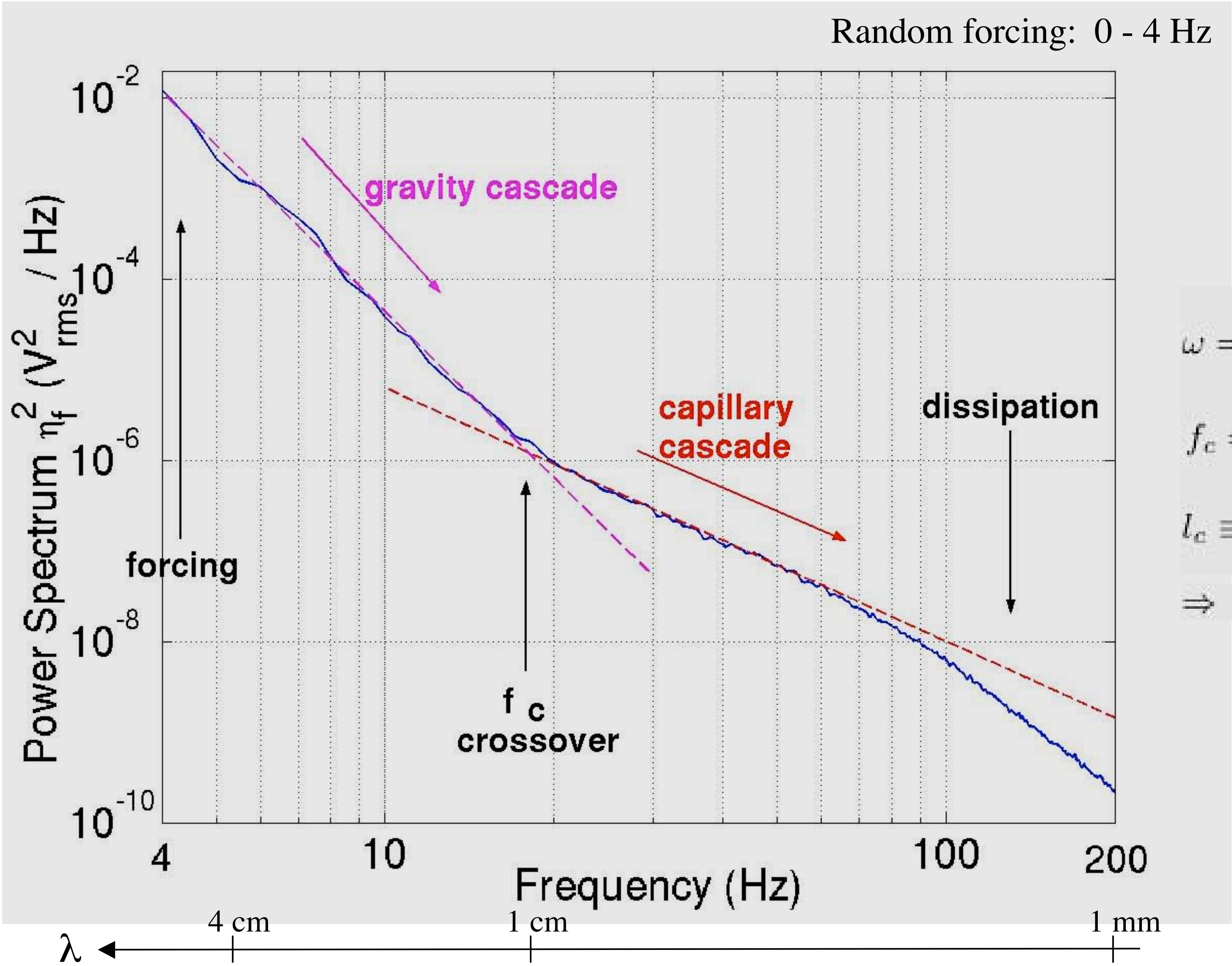


Temporal evolution of the wave height



Height measurement with a capacitive sensor (from 10 μm to 2 cm)

Power spectrum of η(t)



$$\omega = \sqrt{\left[gk + \frac{\gamma}{\rho}k^3\right]}$$

$$f_c = \frac{1}{\pi}\sqrt{\frac{g}{2l_c}} \simeq 17\ Hz$$

$$l_c \equiv \sqrt{\frac{\gamma}{\rho g}} = 1.7\ mm$$

$$\Rightarrow\ \lambda_c \simeq 1\ cm$$

Power law spectra : capillary ~ $f^{-2.8}$; gravity ~ $f^{-6.2}$

Crossover : f_c ~ 20Hz

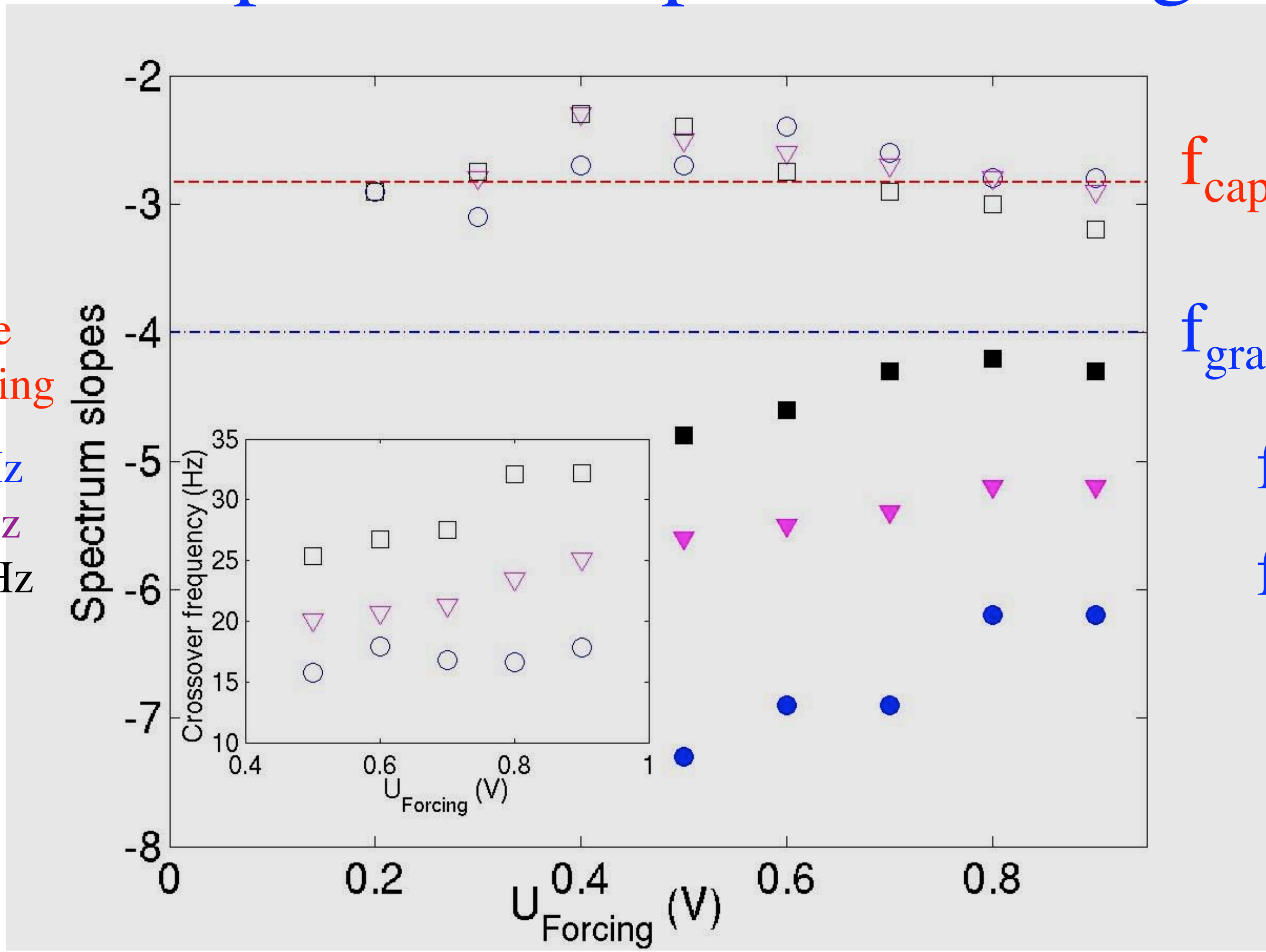
Spectrum slopes vs. forcing

Width of the random forcing

(o) 0 ⇒ 4 Hz

(∇) 0⇒ 5 Hz

(□) 0 ⇒ 6 Hz



Theories:

$f_{cap}^{-17/6}$

f_{grav}^{-4} -Zakharov 1967 -Kuznetsov 2004

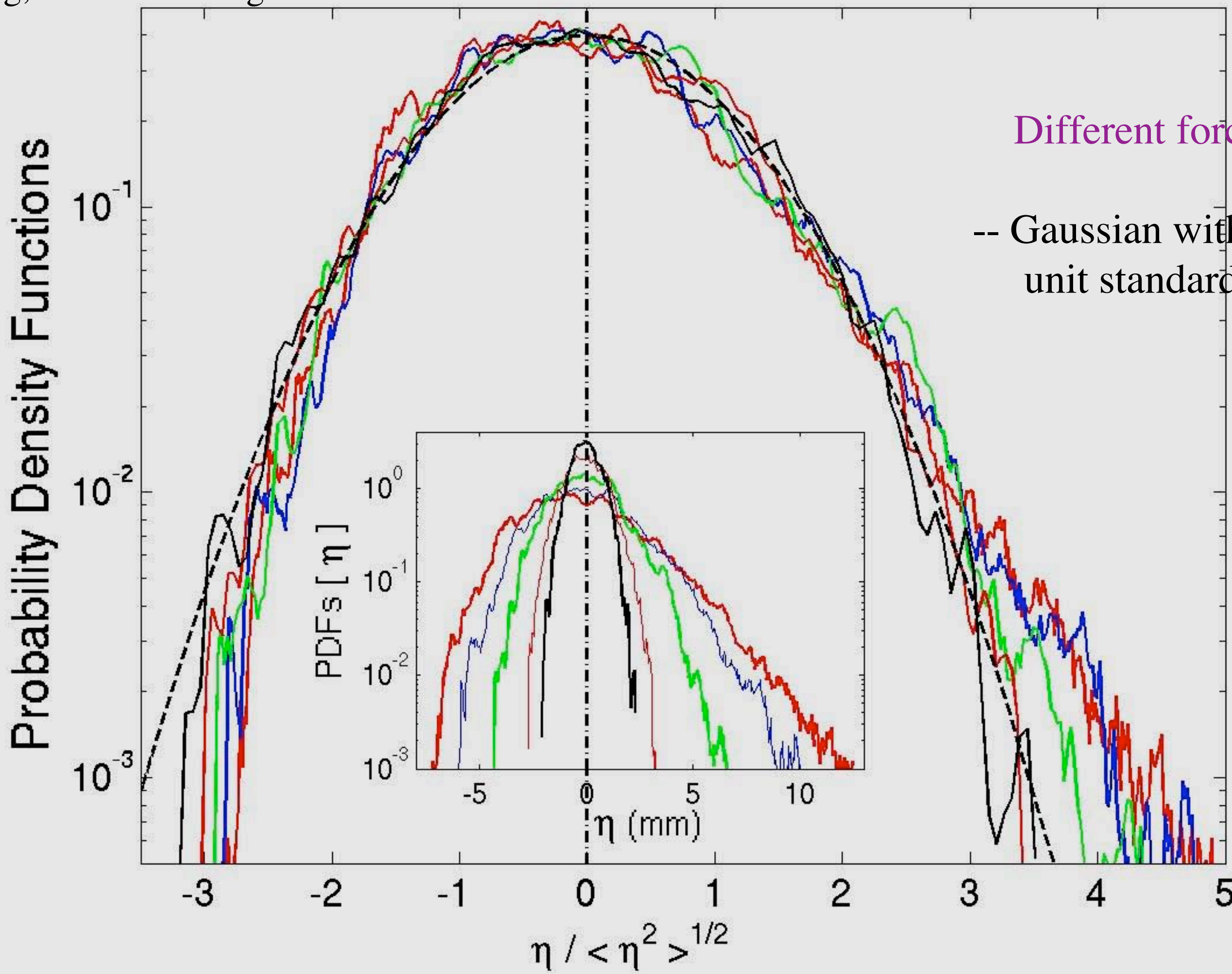
f_{grav}^{-5} Phillips 1958

f_{grav}^{-6} Nazarenko 2006

- Rough agreement for capillary waves (open symbols) with weak turbulence theory (3 waves mixing - Zakharov et al. 1967)
- Depends on the forcing parameters for gravity waves (full symbols) Denissenko et al.2007
- Crossover depends also on the random forcing amplitude and width

Probability density function of the wave amplitude η

Hg, random forcing 0 - 4 Hz



Different forcing amplitudes

-- Gaussian with zero mean and unit standard deviation.

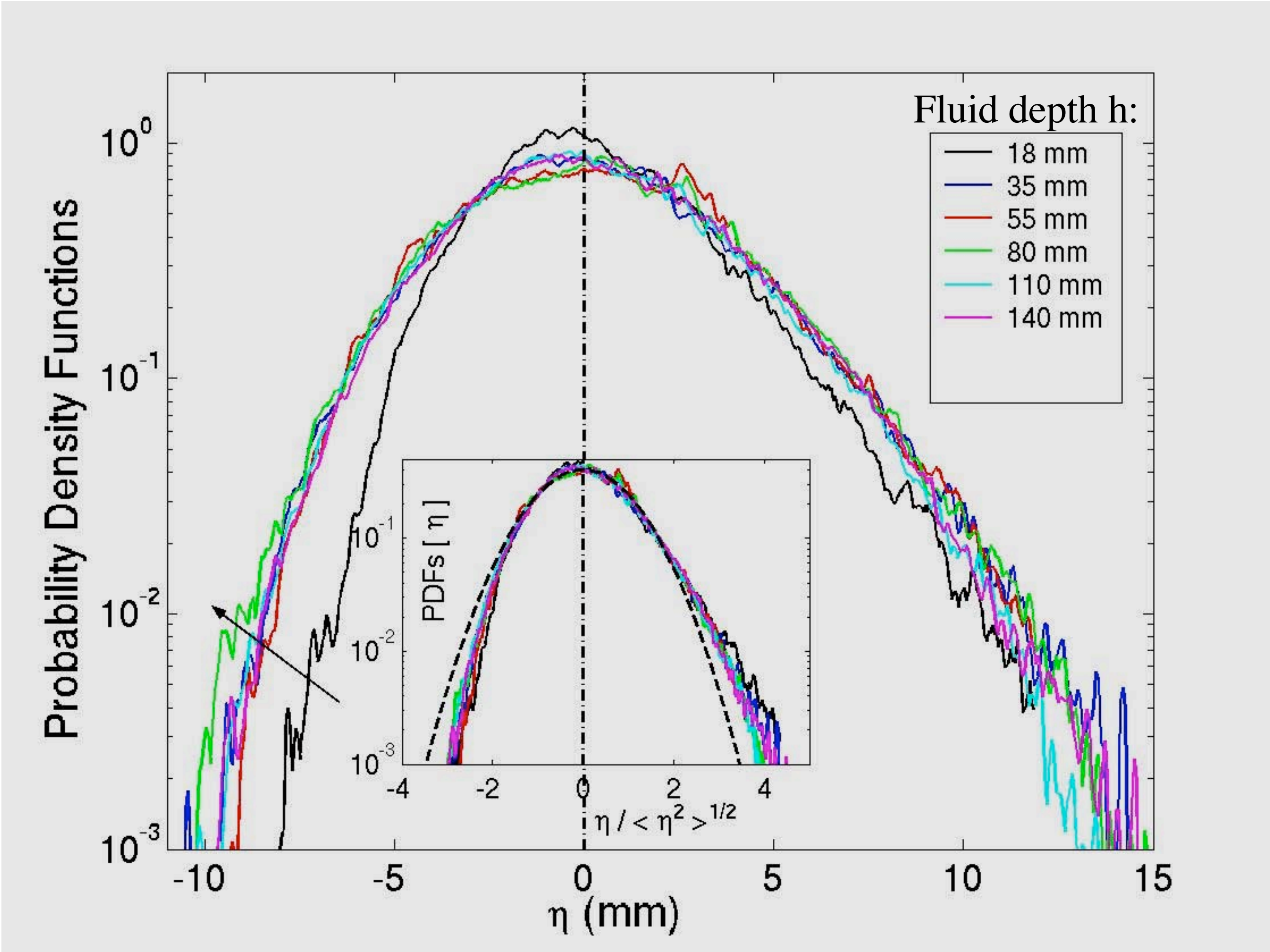
$< \eta > \approx 0$

$0 \leq S \leq 0.7$

- PDFs are asymmetrics
- Non-Gaussian at high enough forcing

Similar with water

Fluid depth effect on the wave amplitude PDF



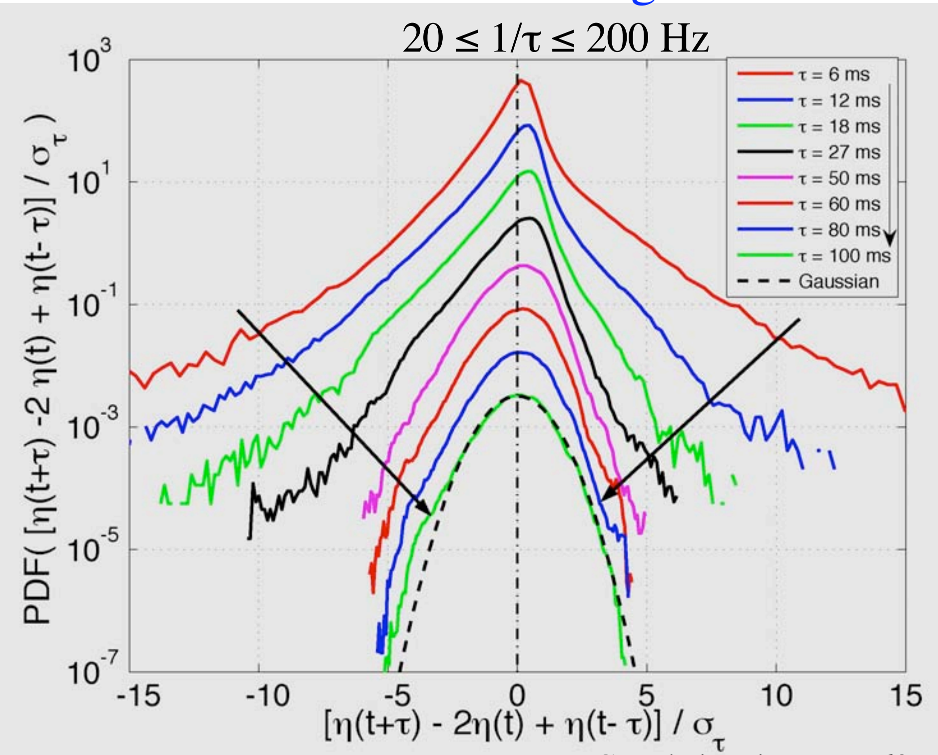
- Asymmetry persists when h >> η
- Asymmetry enlarged when h ≈ η ⟹ Non-Gaussian shape

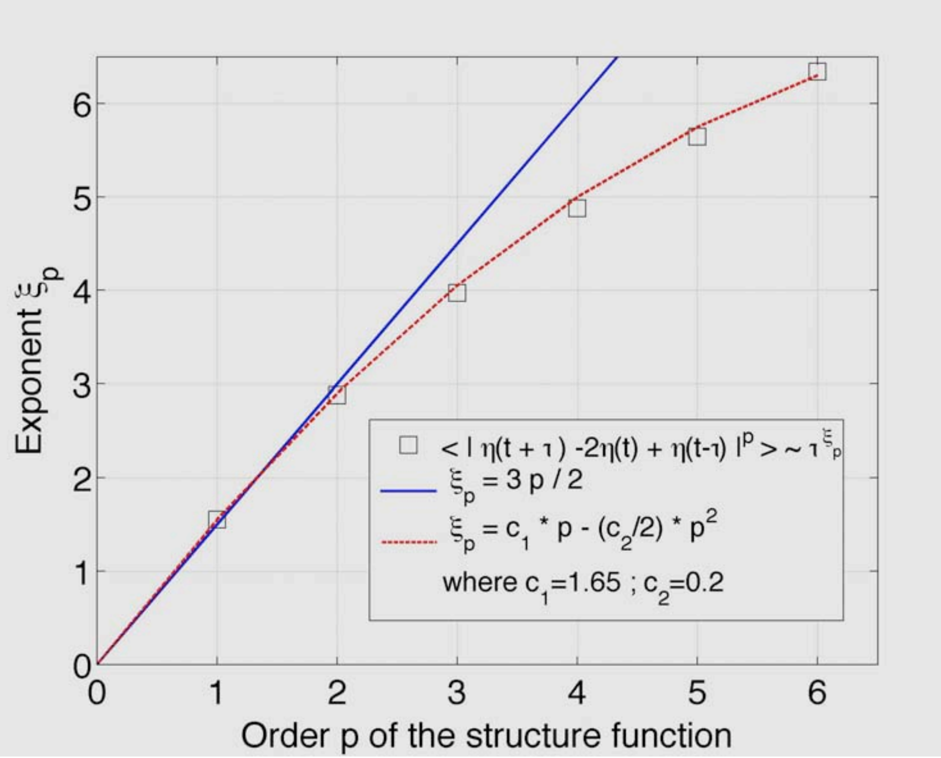
Roughly collapse with the reduced variable η / σ_η

Intermittency in Wave Turbulence

PDF of local slope increments
over a time lag τ

$$S_p^{(1)}(\tau) \equiv \left\langle \left|\eta(t+\tau) - \eta(t)\right|^p \right\rangle$$





Correlation time: $\tau_c \sim 63$ ms

$$S_p(\tau) \sim \tau^{\xi_p}$$

Shape deformation of PDF

with the time lag $\tau < \tau_c$

with ξ_p a nonlinear function of p

For power spectra $E(\omega) \sim \omega^{-n}$ with $n \geq 3$, the statistics of 2nd order increments is relevant

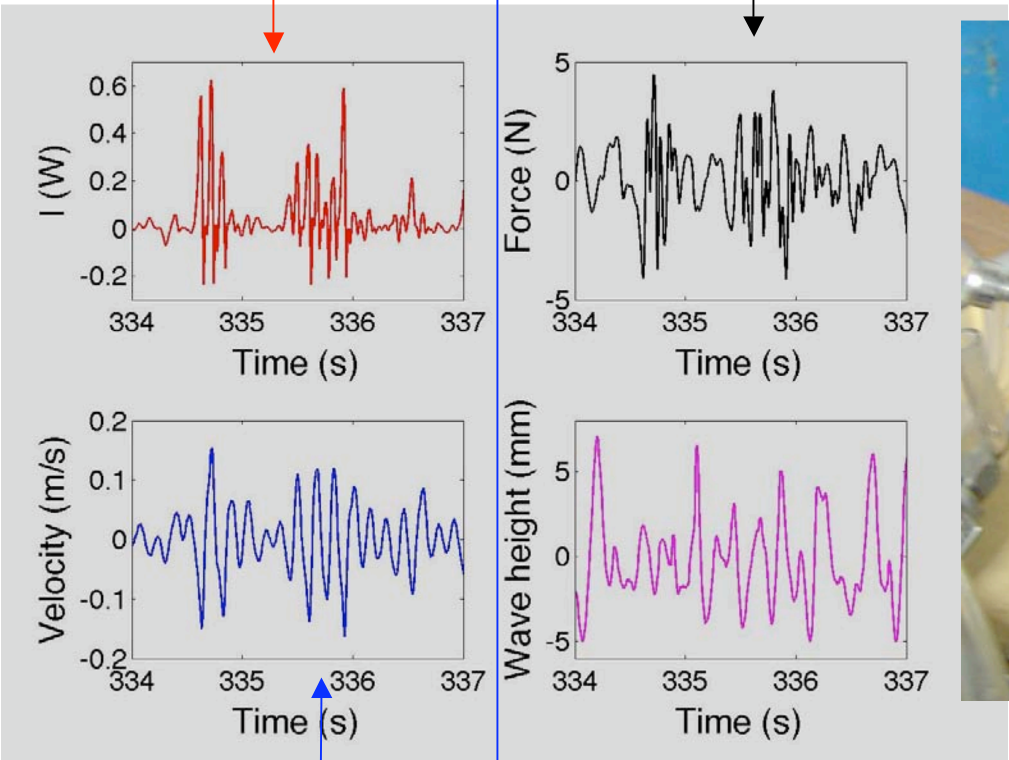
INTERMITTENCY → Coherent structures (cusps + ripples) ? ≠ whitecaps
→ Fluctuations of energy flux ?

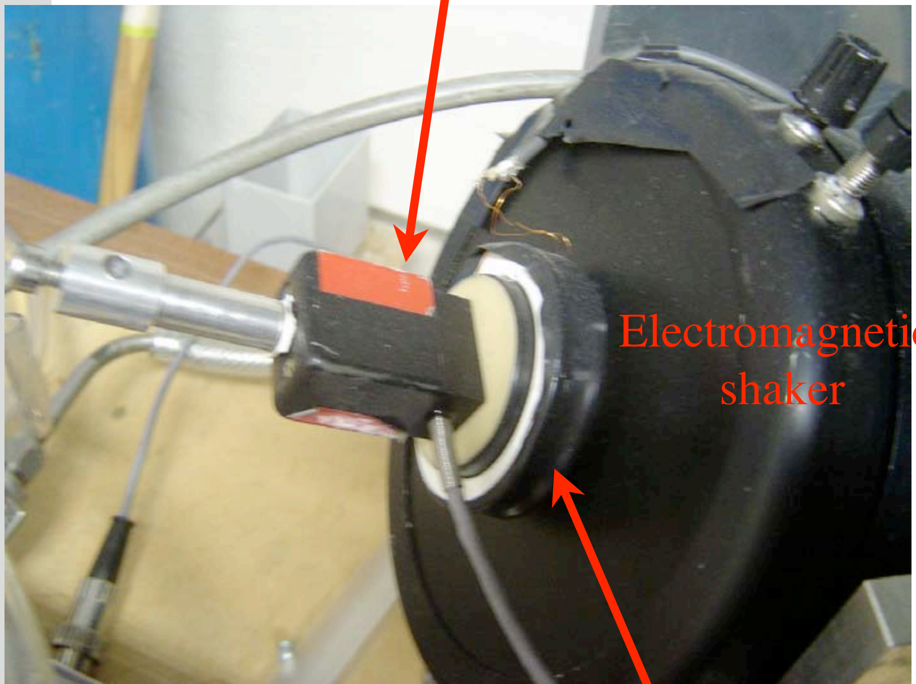
II. Fluctuations of Injected Power

$$I(t) = F(t).V(t)$$

I(t) = F(t)xV(t)
analogic product

Force F applied to the wavemaker
(piezoresitive sensor)



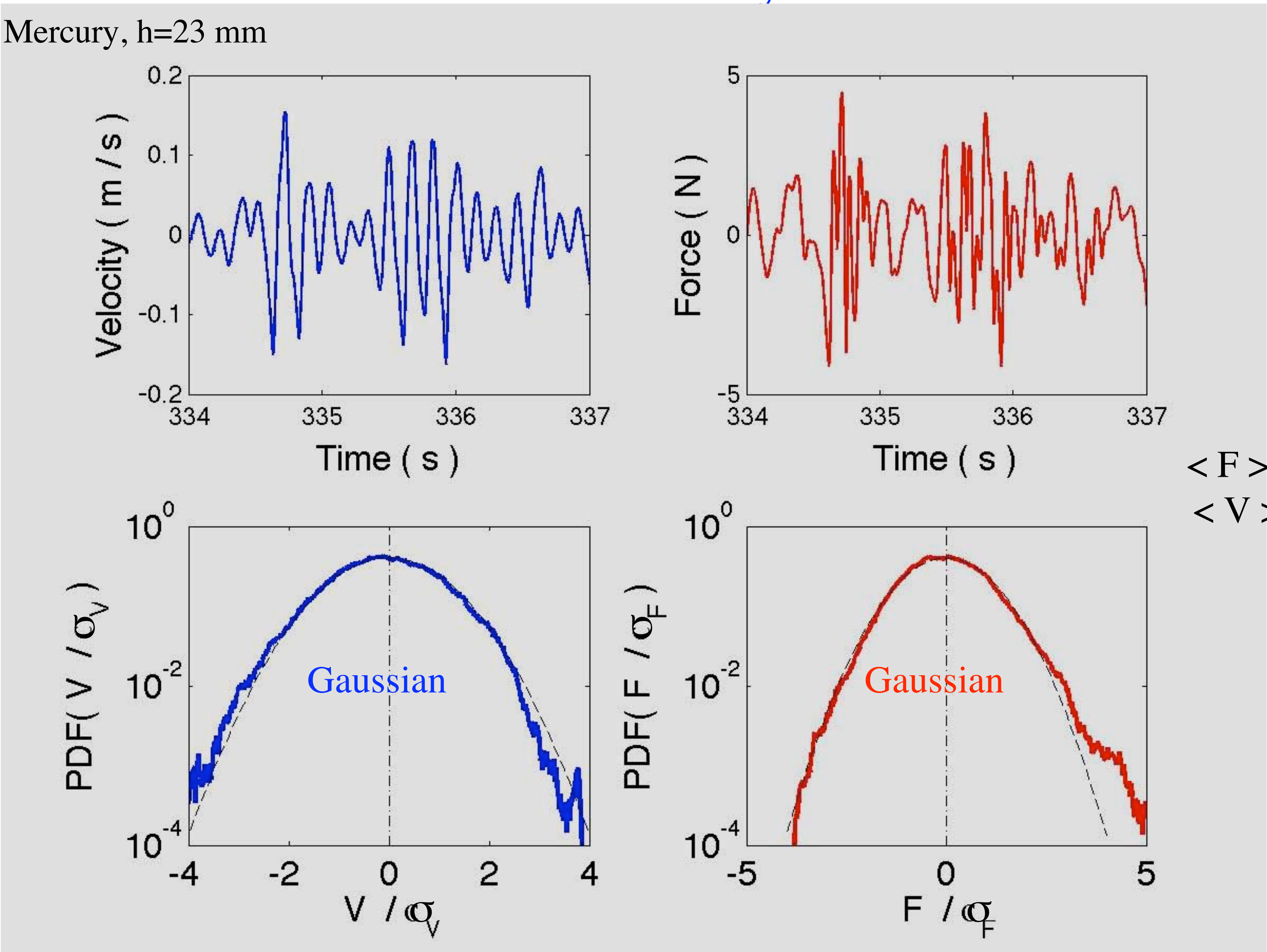


Electromagnetic shaker

Wavemaker velocity V: inductive coil

Contrary to V(t) or F(t),
I(t) consists of strong bursts & quiescent periods with both > 0 or < 0 values

PDF of Force and Velocity of the wavemaker



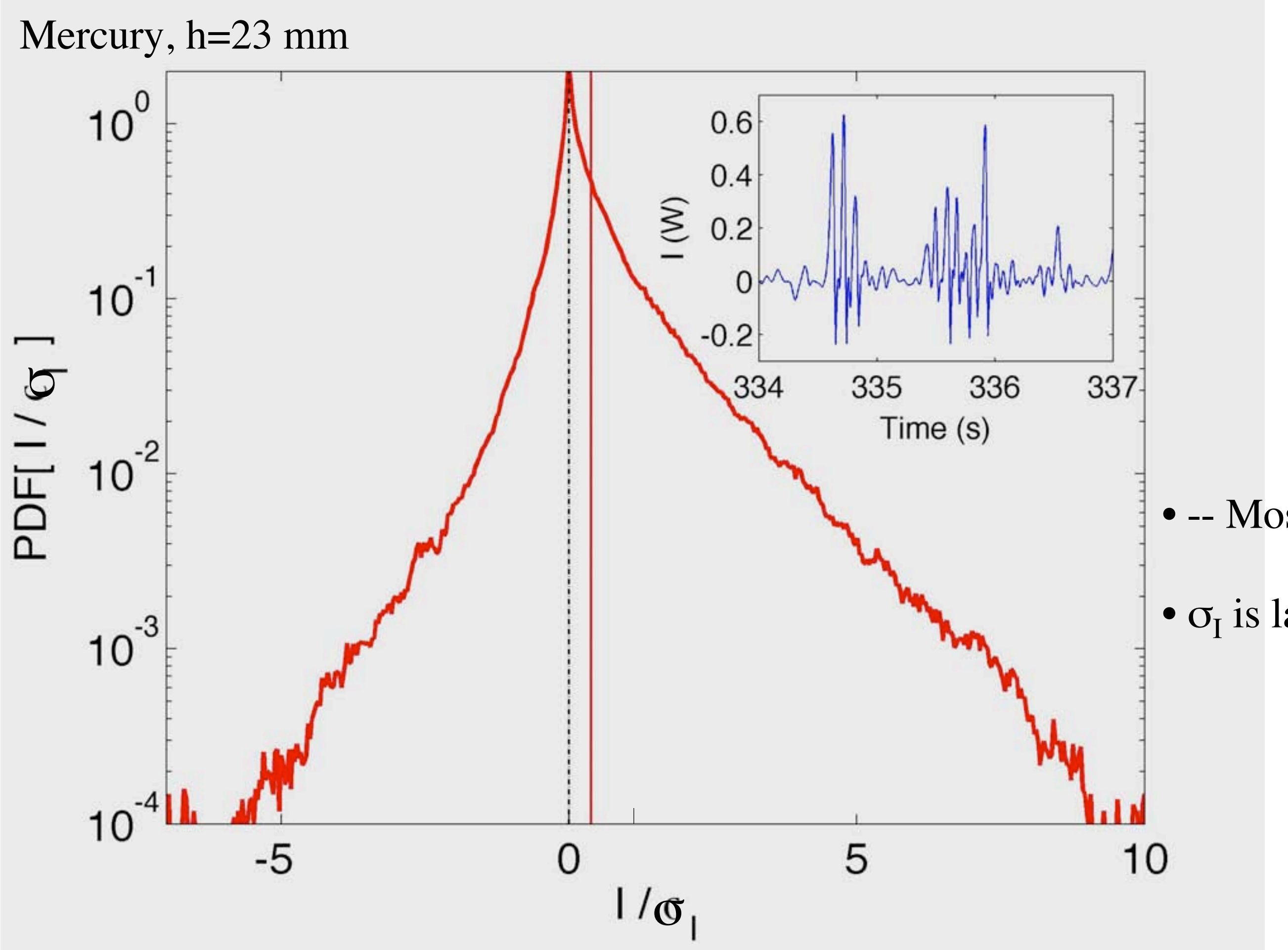
$< F > \approx 0$

$< V > \approx 0$

Both PDF are Gaussian with zero mean value

What about the product of two Gaussian PDFs?

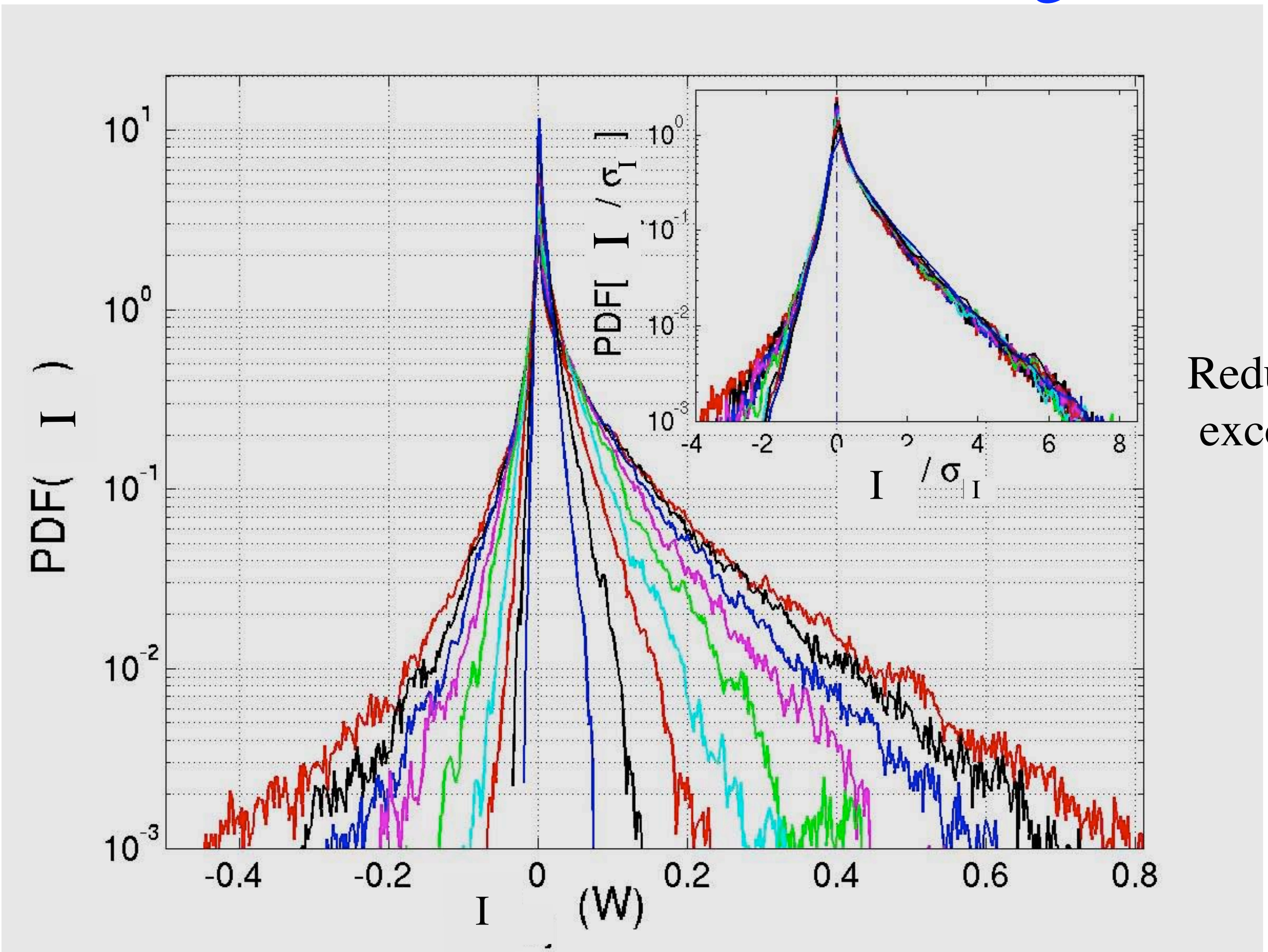
PDF of Injected Power into the fluid



- -- Most probable value of I is 0
- σ_I is larger than $< I > \approx 30$ mW (—)

- PDF (I = FxV) is asymmetric with roughly two exponential tails
- Numerous negative events ⇒ **the fluid gives back some amount of energy to the wavemaker**
- Strong fluctuations ⇒ Rare events visible up to $10\sigma_I$!
- $< I >$ is chosen by the system itself and not by the operator (who drives σ_v)

PDF of I with increasing forcing



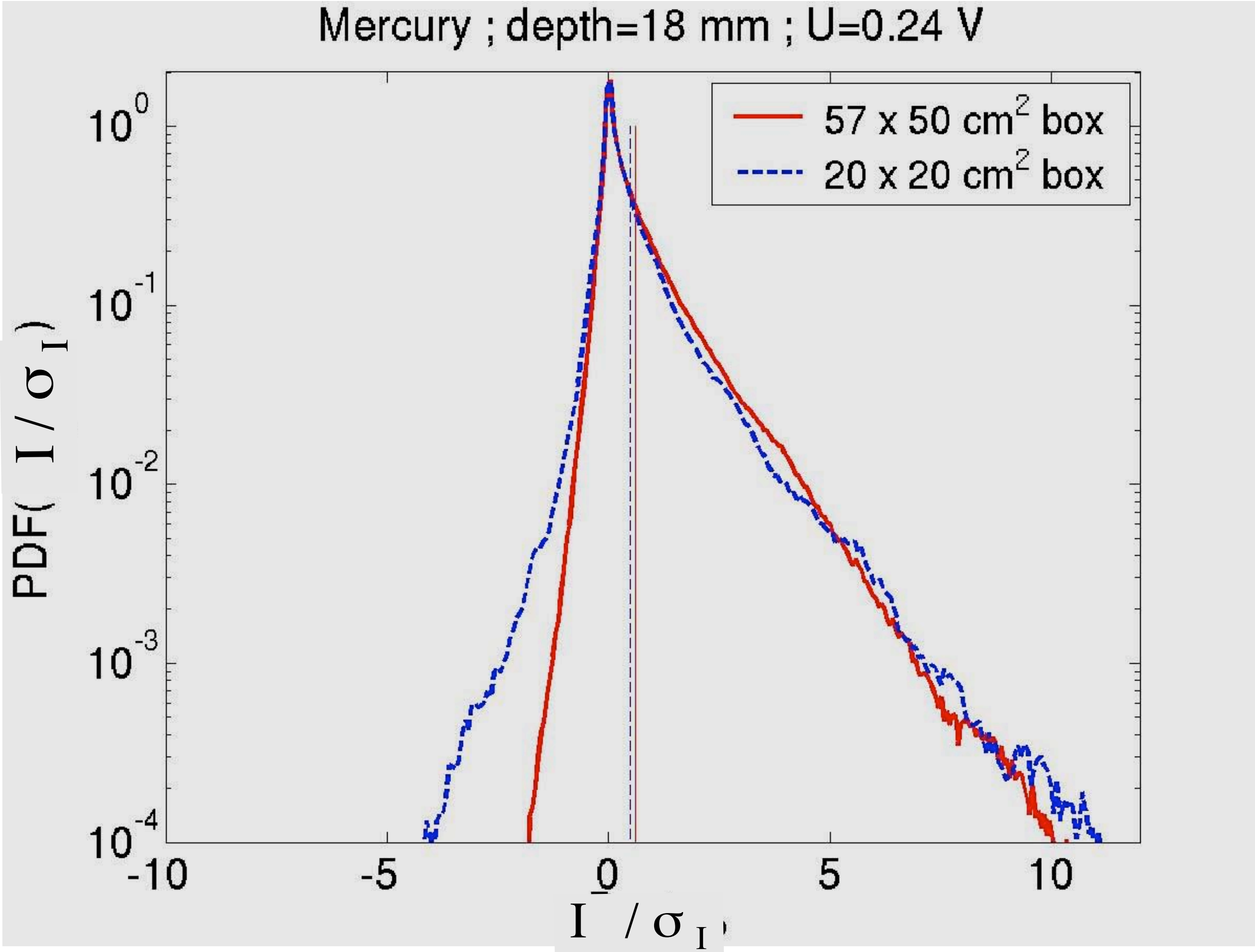
Reduced PDFs roughly collapse except at large negative events

Typical shape of the PDF of injected power**:**

⇒ Relevant in other externally driven dissipative system (see S. Fauve's talk at Warwick)

Roughly similar distribution of I: Cusps I ⇒ 0 ; Exponential tails; Correlation-driven asymmetry

Container size effect on the PDF of the Injected Power



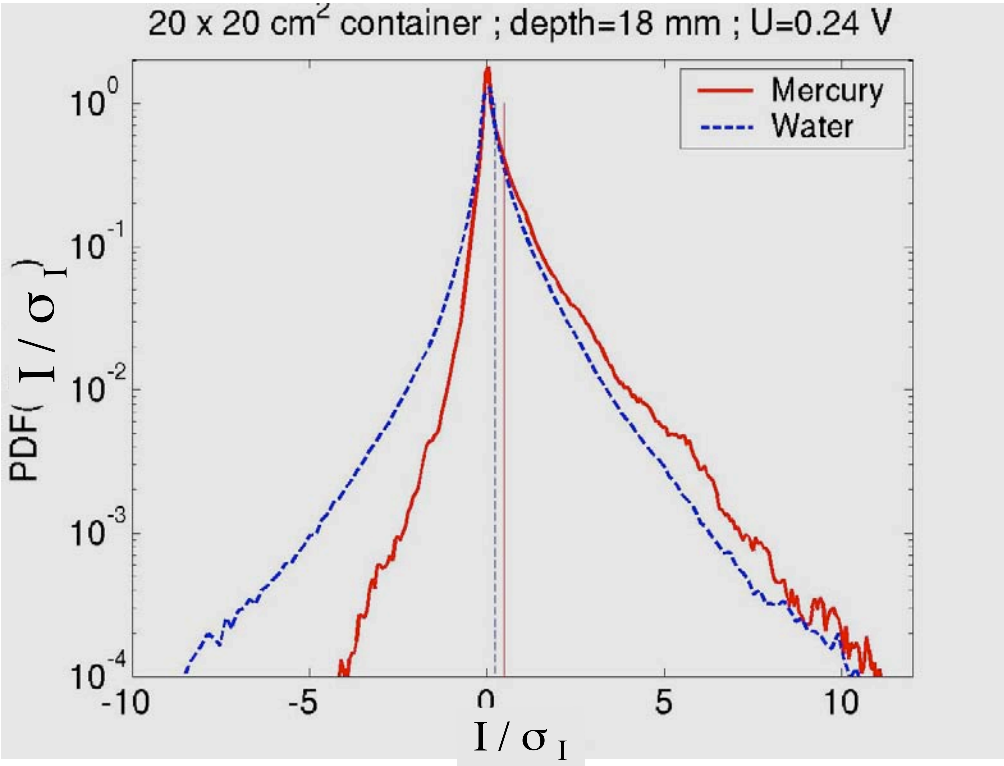
Probability of negative events strongly decreases when the box size is increased

Wave reflected by the boundaries and coherently driving the wavemaker are thus less probable when the box size increases

Fluid effect on the PDF of the Injected Power

$$\frac{\langle I_{Hg} \rangle}{\langle I_{H_2O} \rangle} \propto \frac{\rho_{Hg}}{\rho_{H_2O}}$$

$\Rightarrow$ < I > in water is 14 times lesser than in mercury



— $< I_{Hg} > \sim 22$ mW

-- $< I_{H2O} > \sim 1.6$ mW

PDF(I) in water are more symmetric than in mercury ! $\nu_{H20} >> \nu_{Hg}$

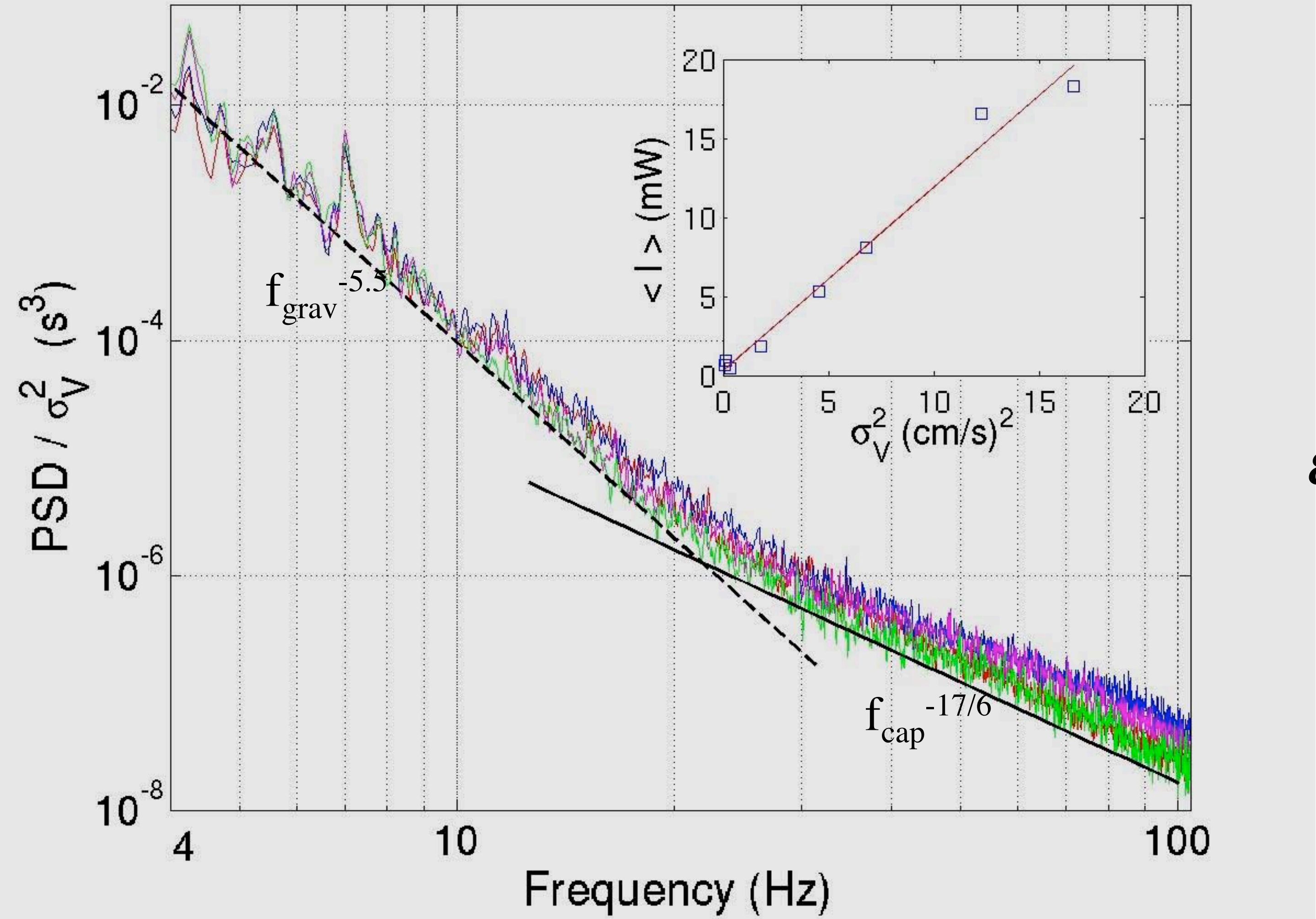
Related to the damping time scale of the wavemaker / the one of the wave field

$\tau_D = M_w \sigma_V^2 / <I> \sim 1/\rho$

water: long τ_D (weak mean dissipation)

mercury: shorter τ_D (strong mean dissipation)

Spectrum scaling with the mean energy flux



$$\sigma_V^2 \equiv \langle V^2 \rangle$$

Mean energy flux

$$\varepsilon \equiv \frac{\langle I \rangle}{\rho S_w} = c\sigma_V^2$$

Experimentally

$$\varepsilon << \left(\frac{\gamma g}{\rho}\right)^{3/4}$$

For both regimes: Power Spectra ~ ε^1 ≠ from weak turbulence results ($\varepsilon^{1/2}$ or $\varepsilon^{1/3}$)

* Finite size effects of the container => additional large velocity scale c [ε] : $(L / T)^3$
* Capillary and gravity waves interact each other and are not independent
* Strong fluctuations of the injected power >> < I >

III. Wave Turbulence in Zero Gravity ?

When $g \Rightarrow 0$: - the capillary length $l_c \propto \sqrt{\frac{\gamma}{\rho g}} \to \infty$

- the crossover frequency $f_c \propto \sqrt{\frac{g}{l_c}} \propto g^{3/4} \to 0$

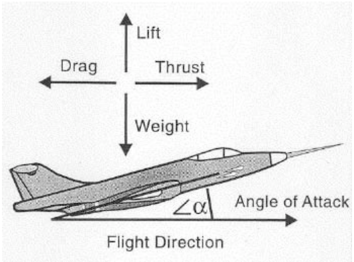
$\Rightarrow$ capillary wave turbulence even for $\lambda >$ cm

QUESTION :

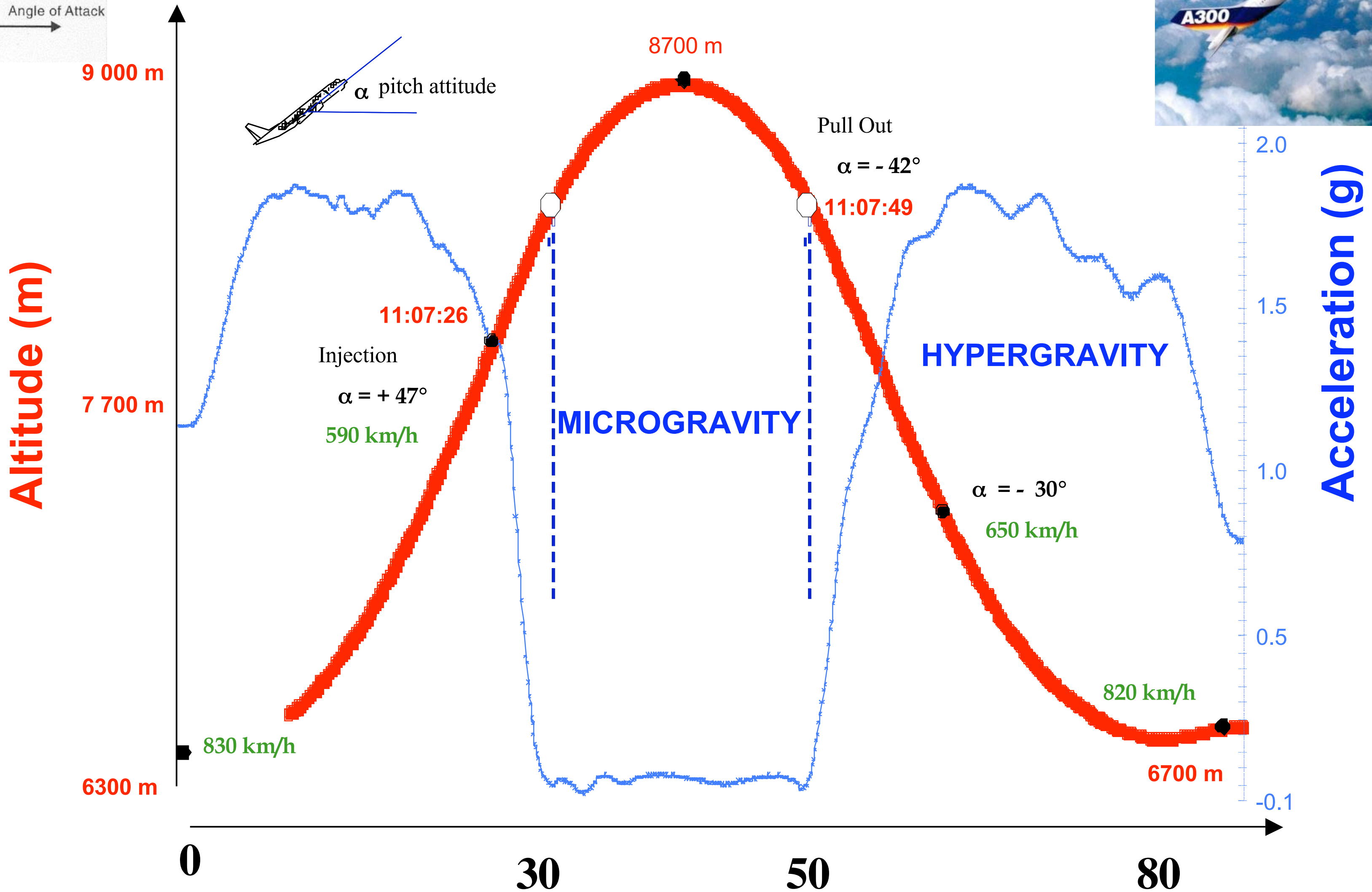
Can we observe and characterize the capillary wave turbulence regime

over a broad frequency range

usually masked on Earth by the regime of gravity waves?



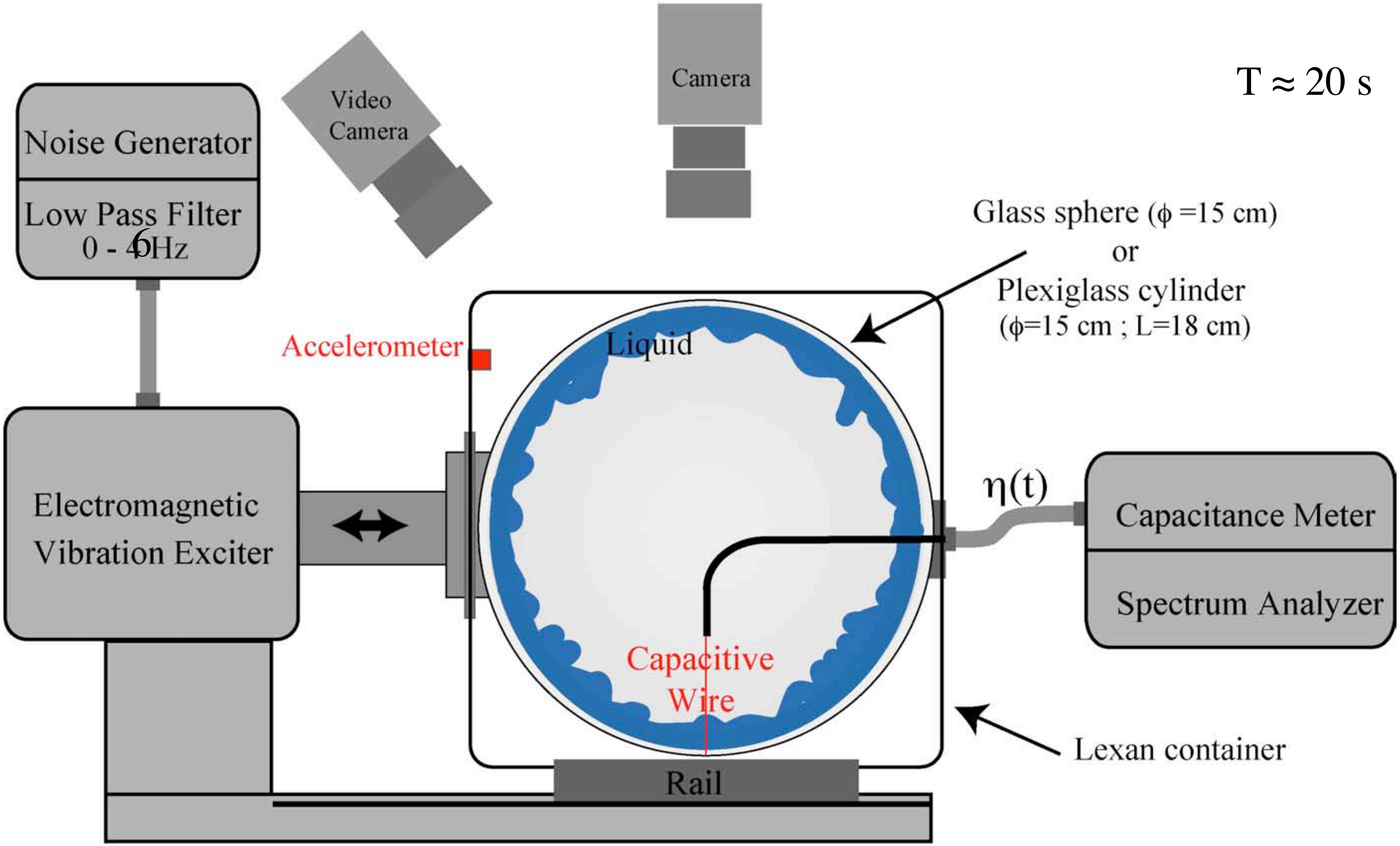
Parabolic flights



Avec l'aimable autorisation de G. Lebarzic

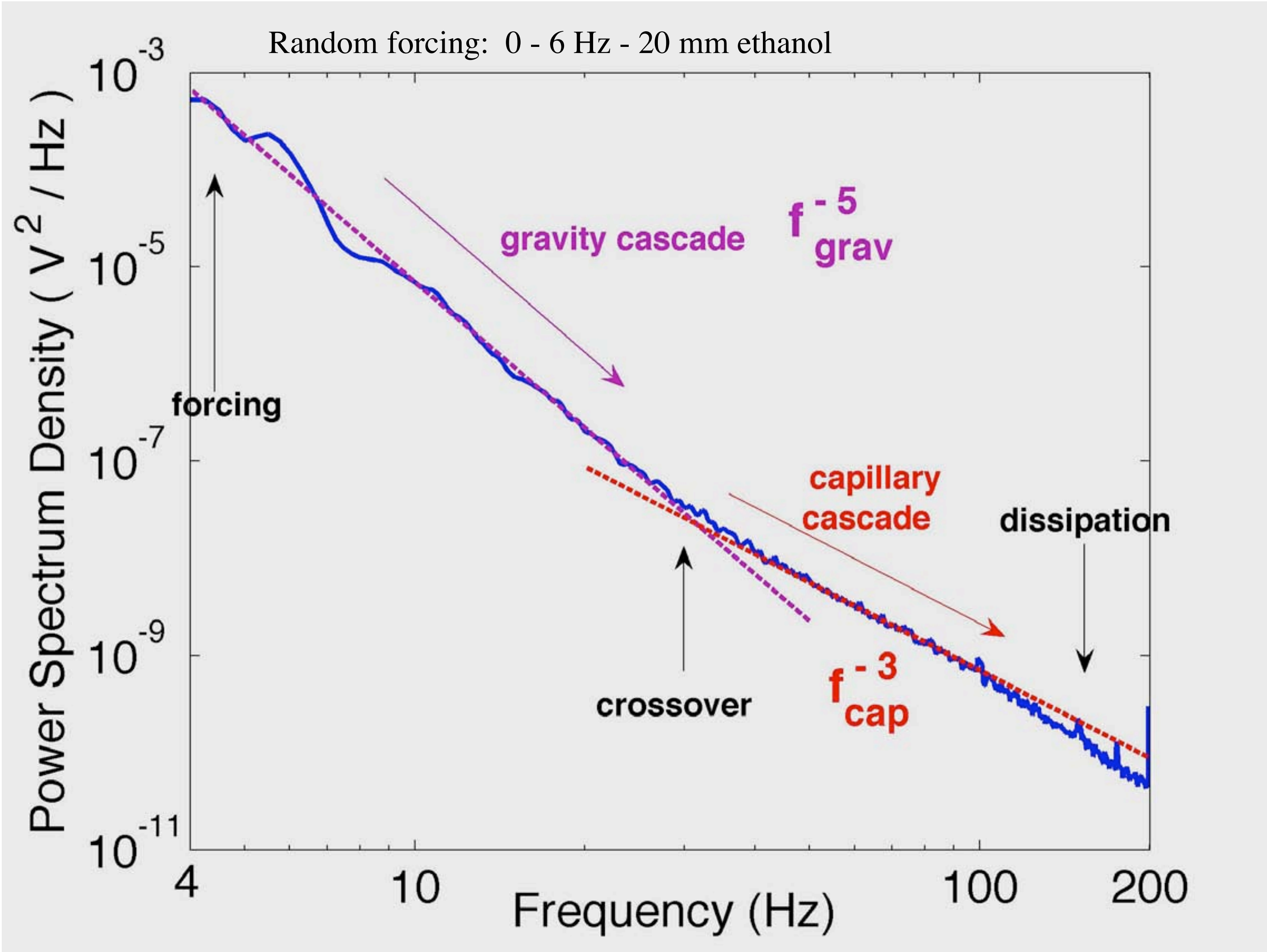
Campagne: 3 flights of 30 parabola at ±0.05g each of 20 s

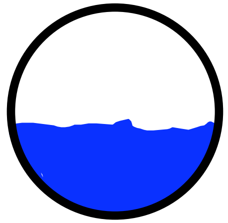
Experimental Setup in Zero Gravity



Water or ethanol : 20 cl (sphere) or 30 cl (cylinder) ⇒ fluid layer of 5 mm depth

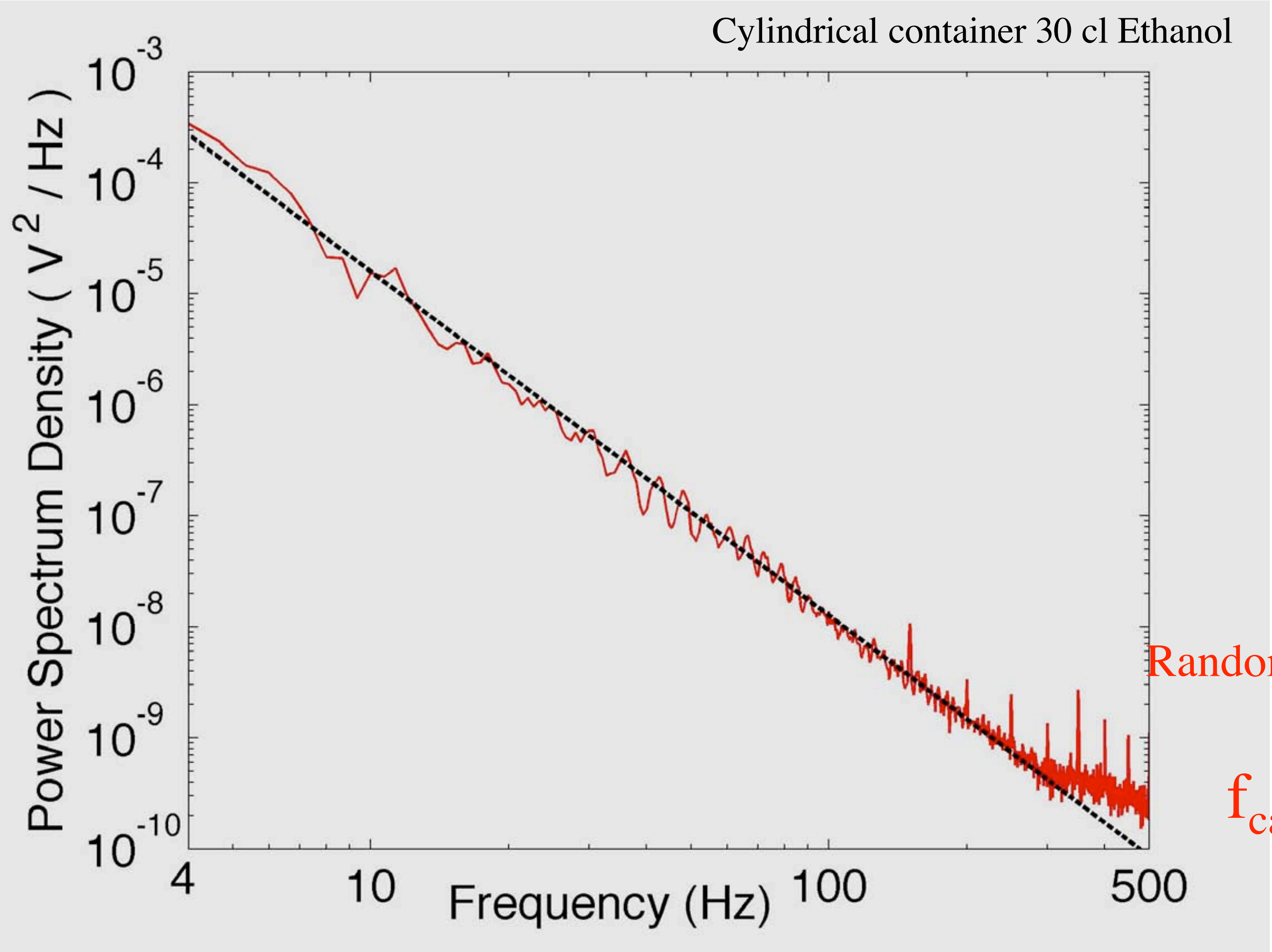
Power spectrum of η(t) with gravity

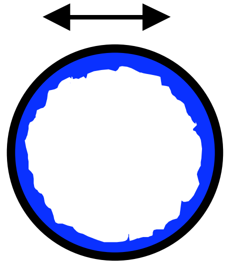




When $g \Rightarrow 0$? Does $f_c \Rightarrow 0$?

Power spectrum of η(t) without gravity





For 0.05g :

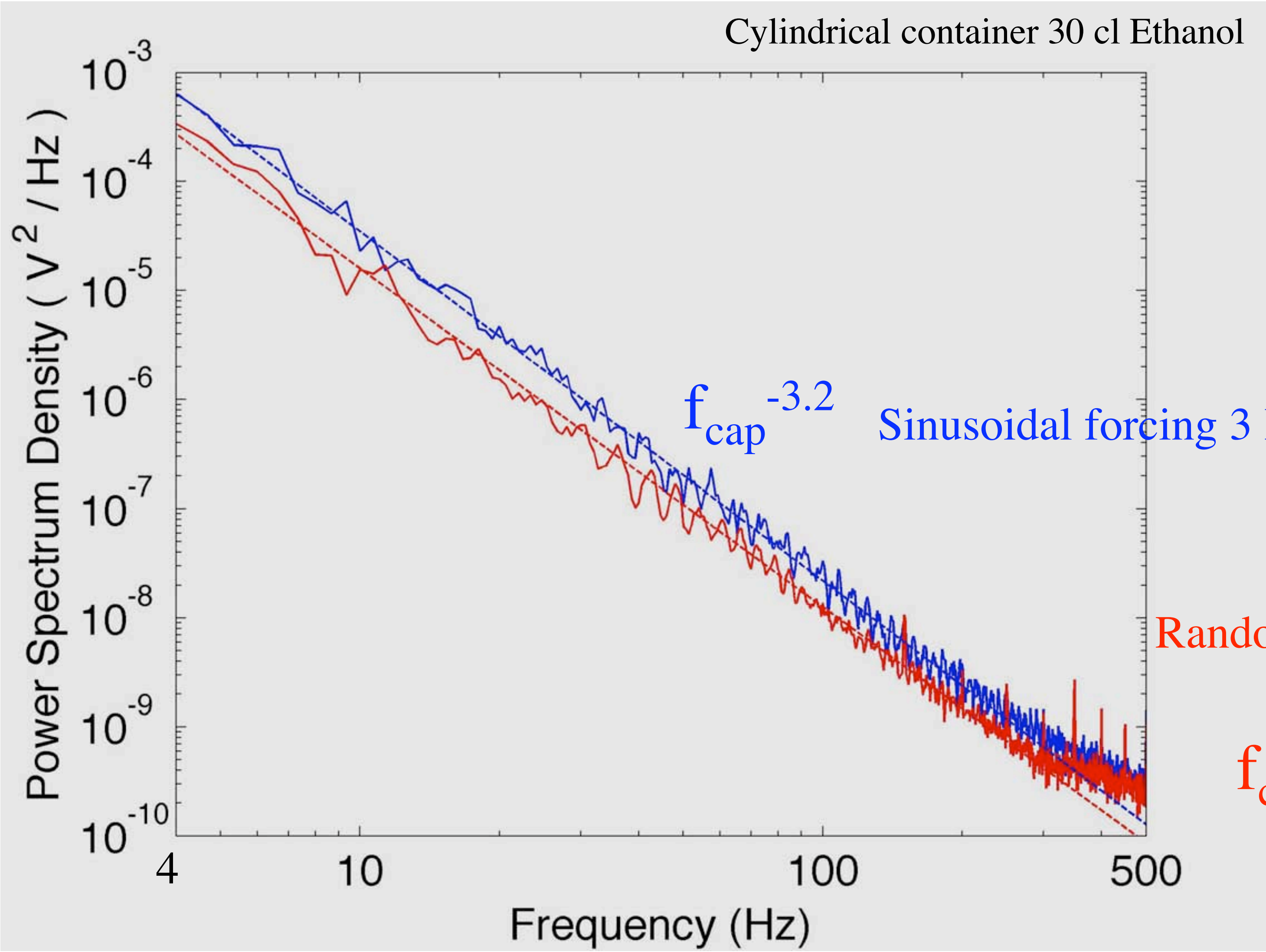
$l_c \approx 2.5$ cm
$\lambda_c \approx 15$ cm
$f_c \approx 0.5$ Hz

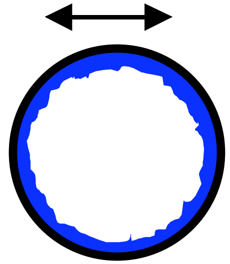
Random forcing 0 - 6 Hz

$f_{cap}^{-3.1}$

Capillary wave turbulence spectrum over 2 decades in frequency!

Power spectrum of η(t) without gravity





Capillary wave turbulence spectrum over 2 decades in frequency!

Spectrum is independent on the large-scale forcing parameter

Wave patterns on a cylindrical fluid surface in zero-g

Cylindrical cell
30 cl of ethanol

↕ f = 30 Hz
a ~ few mm
Γ ~ few g

⇒ stripes

Wave patterns on a cylindrical fluid surface in zero-g

Cylindrical cell
30 cl of ethanol

$\Rightarrow$ hexagons

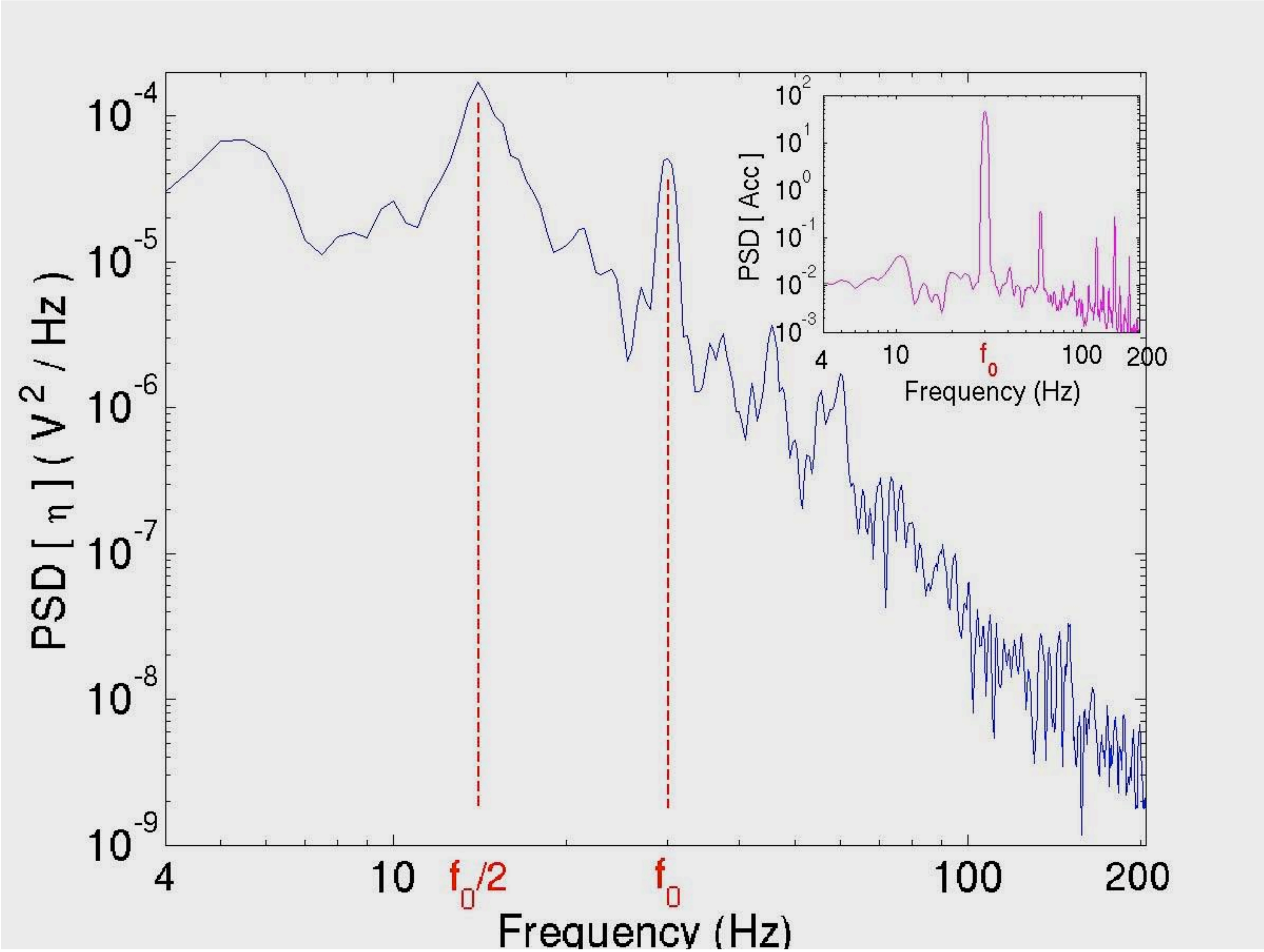
↕ f = 60 Hz
a ~ few mm
Γ ~ few g

Wave patterns on a spherical fluid surface in zero-g

Spherical cell
20 cl of water

f = 60 Hz
a ~ few mm

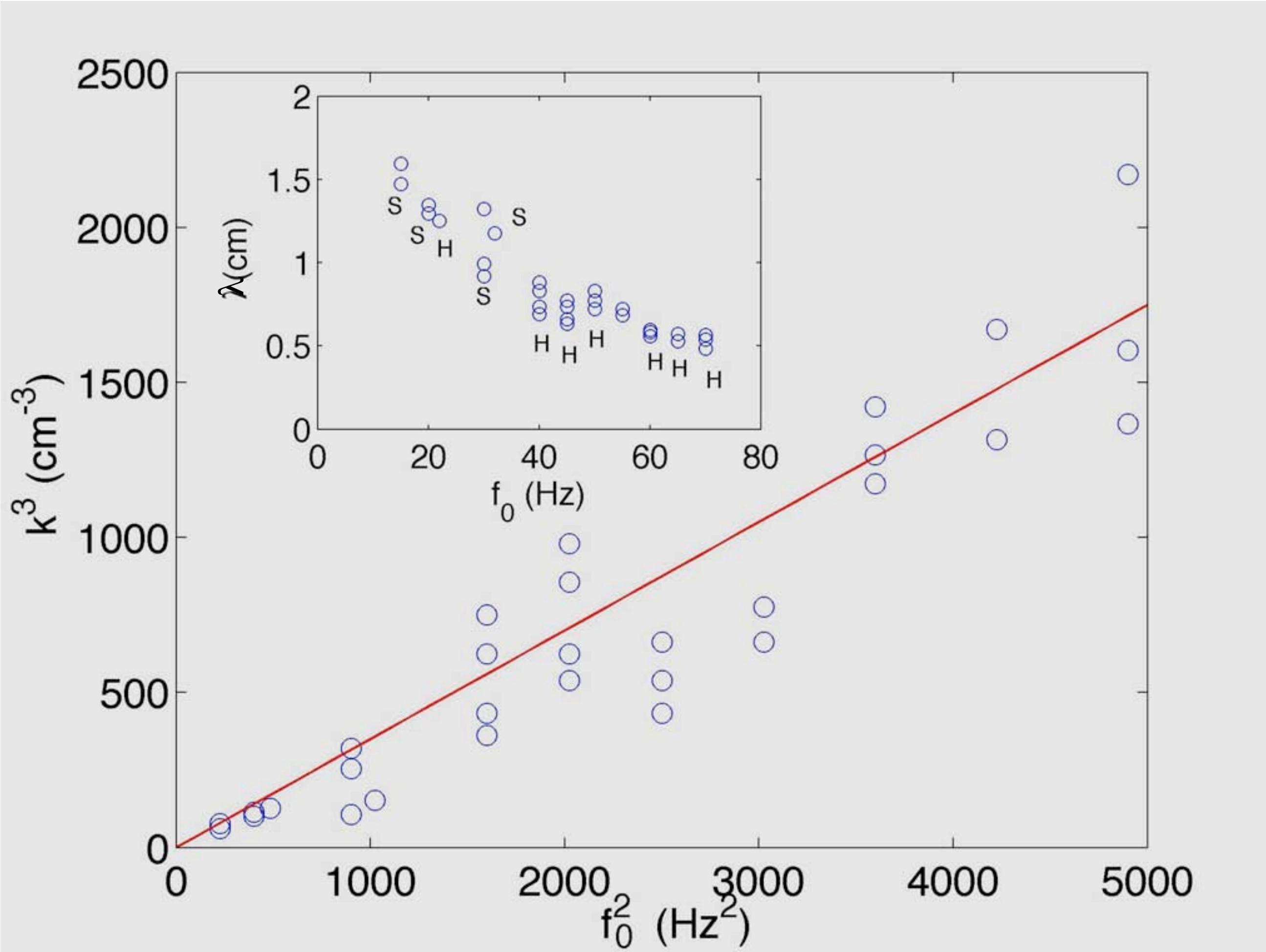
Subharmonic patterns in zero-g



Response freq.: $f_0 / 2$

Excitation freq.: f_0

Pattern wavelengths and dispersion relation in zero gravity



Capillary waves

$\omega^2 = \gamma/\rho\, k^3$

Subharmonic waves

$f = f_0 / 2$

$\omega = 2\,\pi\, f$

$k = 2\,\pi\, /\, \lambda$

↓

$k^3 = \pi^2\, \rho\, /\, \gamma\, f_0^{\,2}$

- Ethanol: 0.35 s^2 / cm^3

• λ decreases with increasing driving frequency f_0

• Measurement of the dispersion relation of linear capillary waves in zero gravity

• Patterns: simple parametric excitation

• More complex dynamics: interplay between sloshing motion and parametric amplification

Conclusions

- Capillary power spectrum - in good agreement with weak turbulence theory
  - over a broad band of frequencies ($g = 0$)

- Non-Gaussian distribution of wave amplitudes
- Measurement of the injected power driving wave turbulence
  - ⇒ Strong fluctuations of the energy flux >> mean value
  - ⇒ Fluid gives back energy to the its driving device
  - ⇒ Shape of PDF(I) relevant in other externally driven dissipative systems

- Intermittency in wave turbulence
  - ⇒ Motivate explanations ≠ Navier-Stokes systems, or coherent structures

- Crossover and gravity spectrum depend on the forcing
- Capillary and gravity power spectrum of $\eta \sim \varepsilon^1$

| ≠ from theories

⇒ Finite size effect of the container,

⇒ Strong fluctuations of I ≠ constant energy flux during the cascade

• E. Falcon, C. Laroche, S. Fauve, Fluctuations of energy flux in wave turbulence, submitted toPRL (2007)
• C. Falcón, E. Falcon, U. Bortolozzo, S. Fauve, Capillary wave turbulence on a fluid surface in zero gravity , submitted toPRL (2007)
• E. Falcon, S. Fauve, C. Laroche, Observation of intermittency in wave turbulence, PRL **98**,154501 (2007)
• E. Falcon, C. Laroche, S. Fauve, Observation of gravity-capillary wave turbulence, PRL **98**, 094503 (2007)

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