# Evidence for upscale energy transfer from the atmospheric boundary layer

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**Abstract**

The longitudinal structure function $S_3(r)$ and the longitudinal and transverse second order structure functions, $S_2^L(r)$ and $S_2^T(r)$ are determined from atmospheric measurements and simulations. Velocities in the tropical boundary layer over the ocean are determined from the average velocity of capillary waves on the ocean surface obtained from satellites using scatterometers (microwave reflections) through an empirical relationship. The simulation is of classical Rayleigh-Bénard convection in a wide box. The observational data finds that $S_3$ is positive in the east-west direction where there is a mean flow and negative in the north-south direction. For the wide box, $S_3(r)$ obeys $-(4/5)\epsilon r$ in the center for $r < d/2$ where $d$ is the height of the box, following the expectation for 3D turbulence. This form is roughly followed for small $r$ even in the boundary layer. However, at larger scales in the boundary layer $S_3(r)$ obeys $+Cr$, as would be expected for 2D turbulence. Similar changes are seen in the second order structure functions.

The Earth

## Two interrelated problems for global dynamics

- Data assimilation
- Reduced models

- I will not directly address either topic.
- What I will address is:
  - A new analysis tool that might aid research in these areas.
  - And an example of how simple, but fully 3D simulations, can be used to inspire new approaches.
- Common problems for data assimilation and reduced models:
  - How does one verify the methods.
  - How to provide long-range scaling statistics to compare to.

### Data assimilation

- If there are gaps in the data, what sort of statistics should the fabricated data obey?
- If particular, what type of statistical correlations should there be over long ranges
  - Within the gap.
  - With known data outside the gap.

### Balanced models

- What kinds of long-range scaling laws (i.e. statistics) should be obeyed?
- Can we get mid-scale atmospheric data with a -5/3 law to compare to?

# Outline

**Mid-latitude stratospheric scaling**

- What we DON'T KNOW from rotating, stratified periodic box calculations.
  - Métais & Herring
  - Bartello
  - Kimura & Herring
  - Carnevale
  - L. Smith & F. Waleffe,
- Recent observational results of Cho and Lindborg from aircraft data.
  - J. Cho & E. Lindborg, *J. Geophys. Res.* **106**, 10223 (2001).
  - E. Lindborg & J. Cho, *J. Geophys. Res.* **106**, 10223 (2001).

**Boundary layer of non-rotating convective layers**

- Thermally drive convective layer: simulations
- Equatorial atmospheric boundary layer: scatterometer and other data.

# Equations of motion

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = \frac{1}{\rho}\vec{\nabla}P + \nu\nabla^2\vec{u} \; + \; \hat{g}\alpha\theta \; - \; 2\Omega \times \vec{u} \qquad \vec{\nabla} \cdot \vec{u} = 0 \tag{1}$$

*dissipation* *buoyancy* *rotation*

$$\frac{\partial \theta}{\partial t} + (\vec{u} \cdot \vec{\nabla})\theta + w\left(\frac{dT}{dz}\right) = \kappa\nabla^2\theta \tag{2}$$

- On large scales in the atmosphere, only the vertical component of vorticity used.
- Usually called quasi-geostropy
- Why does quasi-geostrophy work? (only $\vec{\omega} = (0, 0, \omega_z(z))$ )
- In simulations we believe via:
  - Rotation
  - Stratification
  - Periodic thinness

# Numbers, Lengths, Frequencies

TURBULENCE

Reynolds number

$$Re = \frac{UH}{\nu}$$

Taylor microscale

$$\lambda = \frac{U}{\sqrt{\epsilon/\nu}}$$

and Re number

$$R_\lambda = \frac{U\lambda}{\nu}$$

ROTATION:

Rossby number

buoyancy/rotation

$$L_R = \frac{\sqrt{gH}}{f}$$

deformation radius

$$\frac{U}{Hf}$$

time

$$f^{-1}$$

STRATIFICATION

Froude number

=shear/buoyancy

$$Fr = \frac{U/H}{N}$$

Brunt-Vaisala frequency

$$N^2 = \frac{g}{\theta}\frac{\Delta\theta}{H}$$

Richardson number

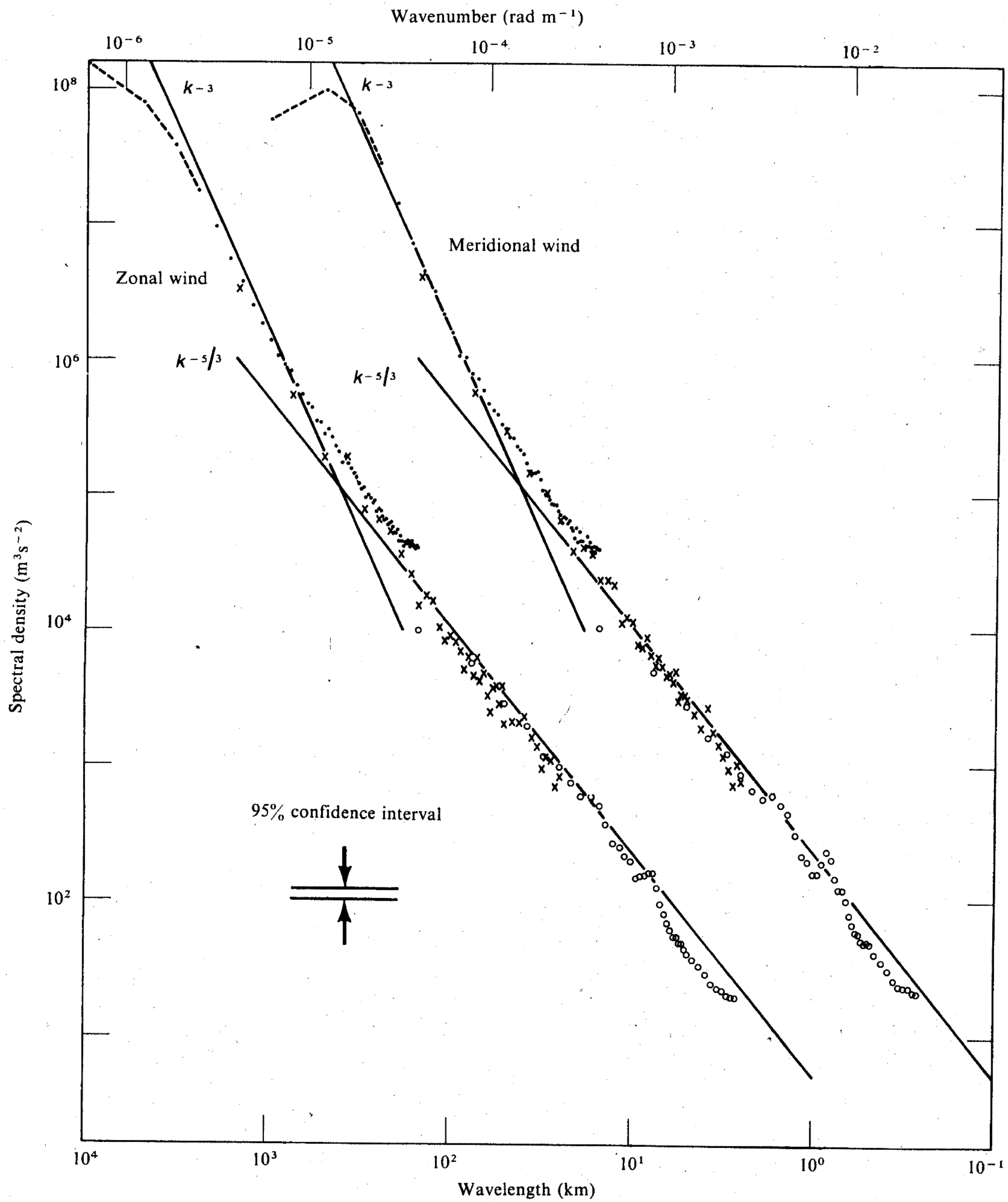
unstable if $Ri < 1/4$

$$Ri = Fr^{-2}$$

CONVECTION:

Rayleigh number

$$Ra = \frac{g\alpha dT/dz}{d^4\nu\kappa} = Re^2 Pr Ri \sim R_\lambda^4$$




Stratospheric spectra from commercial aircraft data from G.D. Nastrom, K.S. Gage & W.H. Jasperson, *Nature* **310**, 36–38 (1984). $k^{-5/3}$ was traditional associated with a 2D upscale transfer of energy and is the source of the butterfly flapping its wings myth. The largest scale $k^{-3}$ regime was interpreted as a downscale enstrophy transfer.

**Sources of 2D**

Why does quasi-geostrophy work? (only $\vec{\omega} = (0, 0, \omega_z(z))$ )

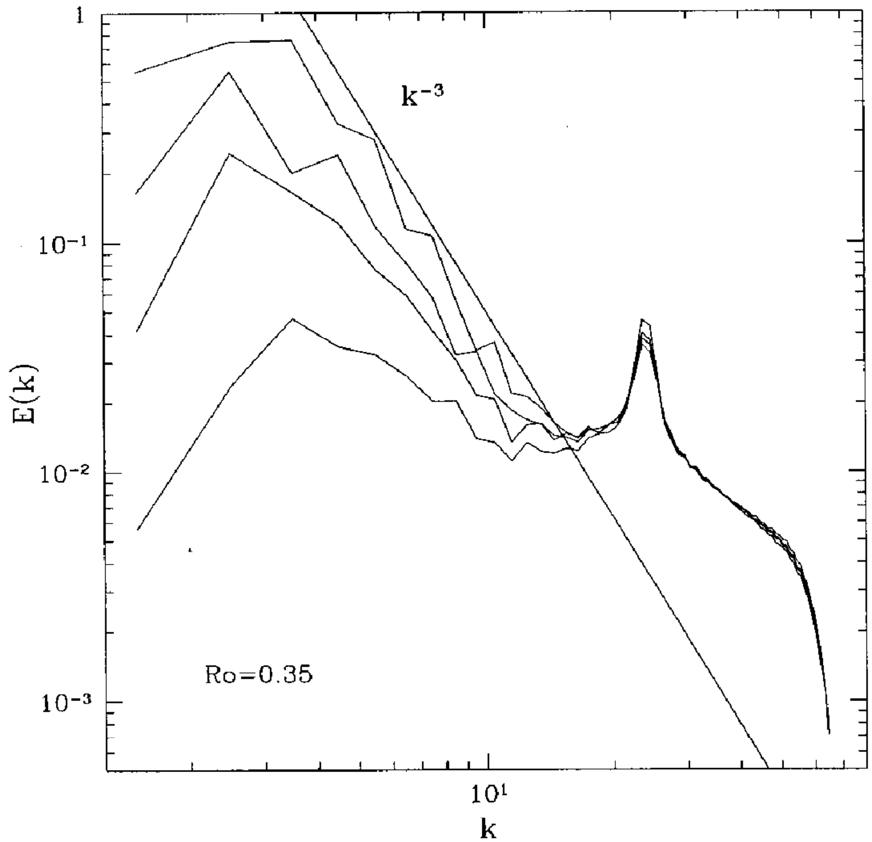
- Rotation
  - Outside the equatorial regions the atmosphere is rotating.
  - Gives quasi-geostrophy in the $Ro \to 0$ limit.
  - But atmosphere is only moderately small $Ro$.
- Stratification
  - Stratosphere is strongly stratified (as its name implies).
  - Gives 2D like vortex motions in layers
  - But strong shears between the layers makes it fully 3D
- Periodic thinness
  - Atmosphere is thin.
  - In the infinitely thin limit gives 2D Navier-Stokes with 3D motions advected (slaved) by 2D motion
  - But periodic thin domains are artificial

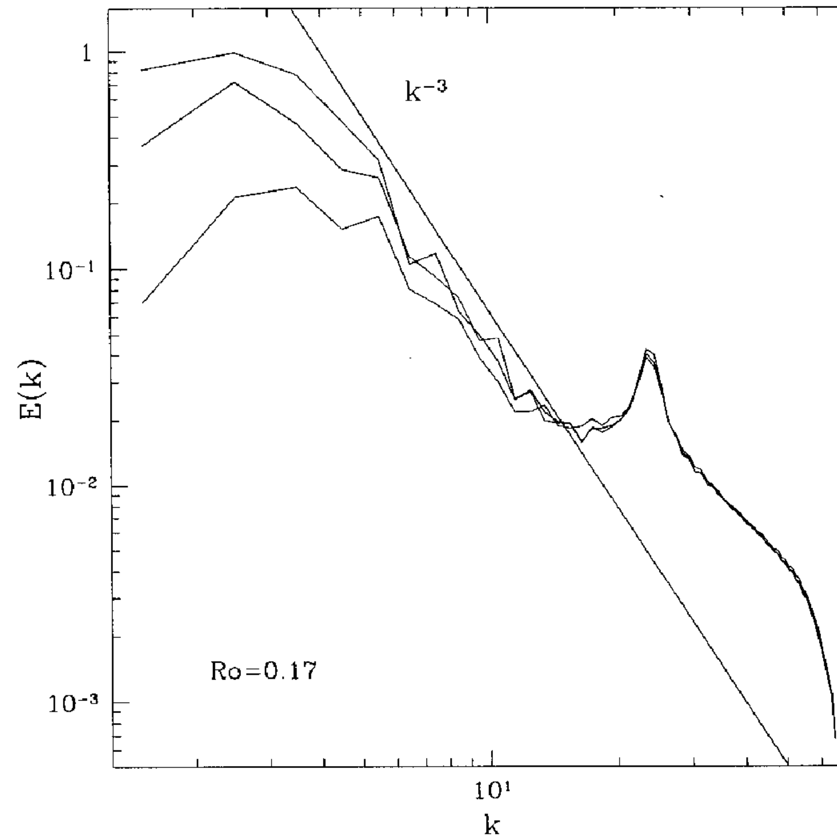
**Sources of quasi-geostrophy**

- Averaging over fast modes eliminates horizontal vorticity components for very strong rotation, very low $Ro$. First Bartello heuristically, then Majda and Embid rigourously.
- Smith and Waleffe: If moderate rotation, the full terms still exist. Can only prevent the occupation of full 3D large scale vorticity states by having a weak, small-scale forcing.
- Upscale energy transfer through rotation will occur only through $1/Ro^2$ terms.
- Will see a upscale energy transfer and a large-scale geostrophic state only for time $\sim 1/Ro^2$.

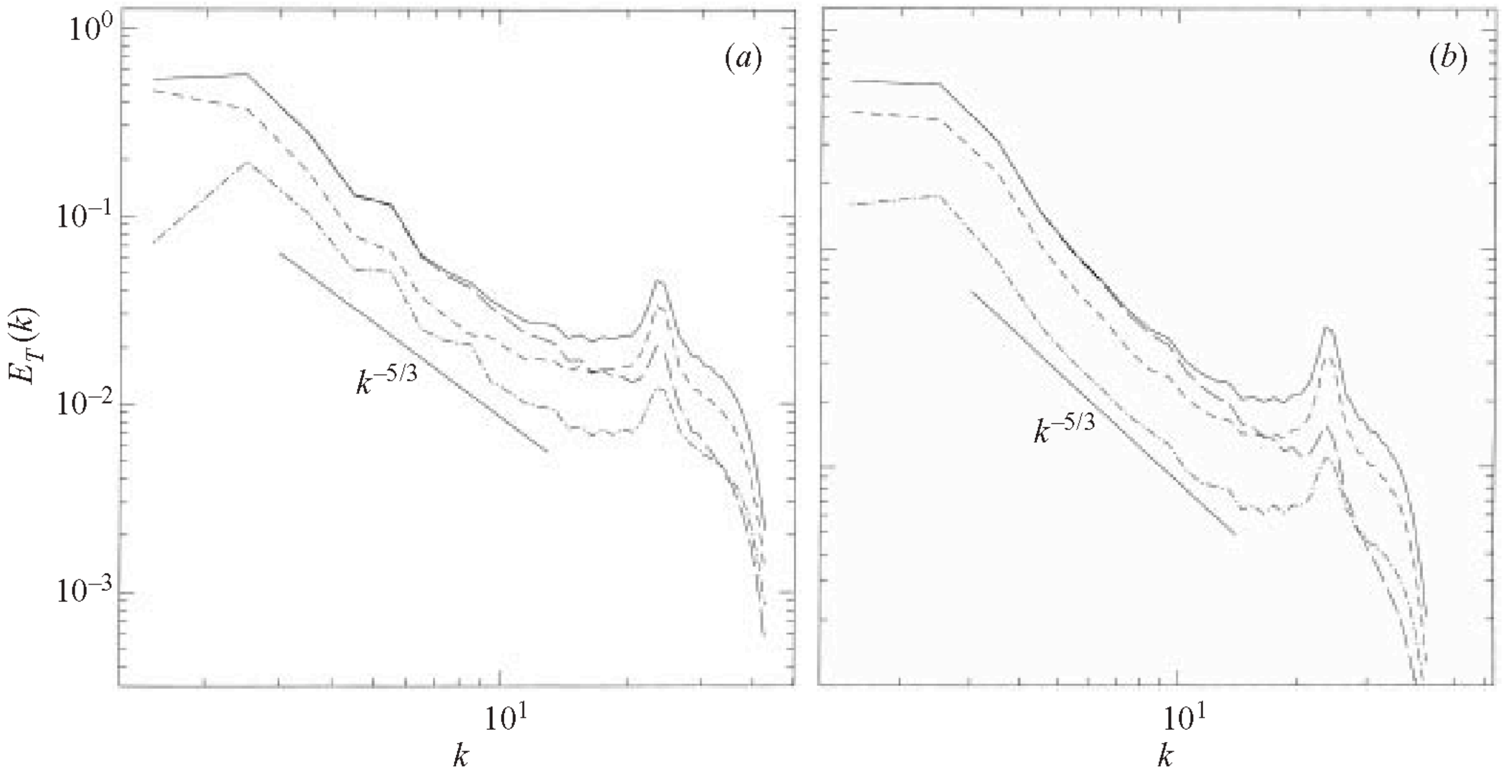
**Effect of stratification**

Strong stratification still yields upscale $k^{-5/3}$ (probably a result going back to Herring and Métais)



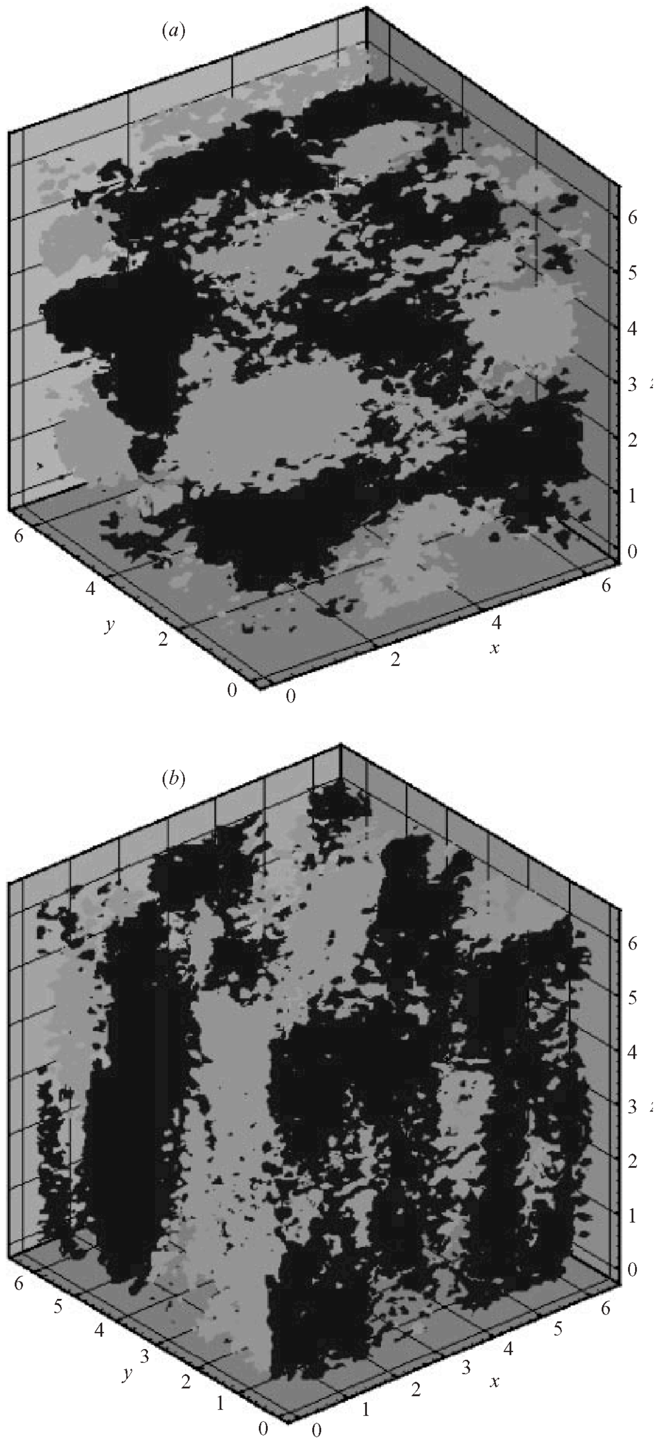




From Waleffe, Phys. Fluids (1999) p5. Full energy sepctra $E(k)$ for $Ro = 0.35$ at $t = 139, 246, 331, 471$; $Ro = 0.17$ at $T = 172, 186, 245$



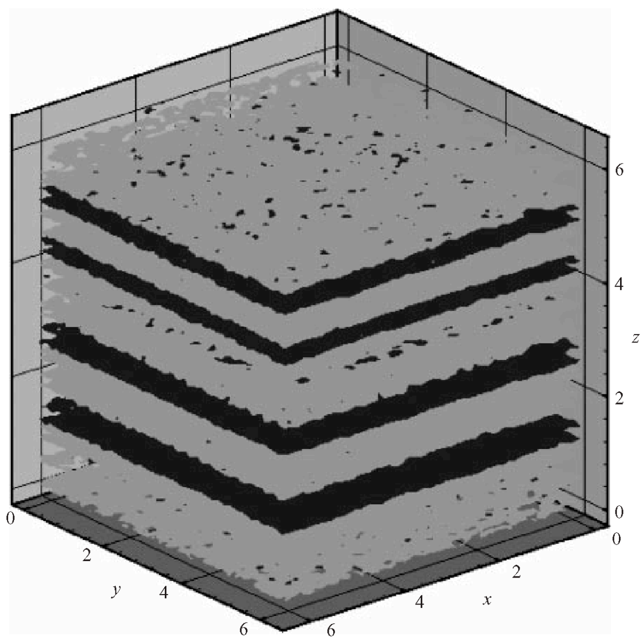

From, Smith/Waleffe, JFM (2002) p15. Total energy vs. wavenumber (solid) for $Fr = 0.21$, and: (a) $Ro = 0.42$ ($N/f = 2$) and $T \approx 170$; (b) $Ro = 0.21$ ($N/f = 1$) and $T = 120$. Total energy in the PV modes (long dashes); kinetic energy (short dashes); potential energy of the density fluctuations (dash-dot)

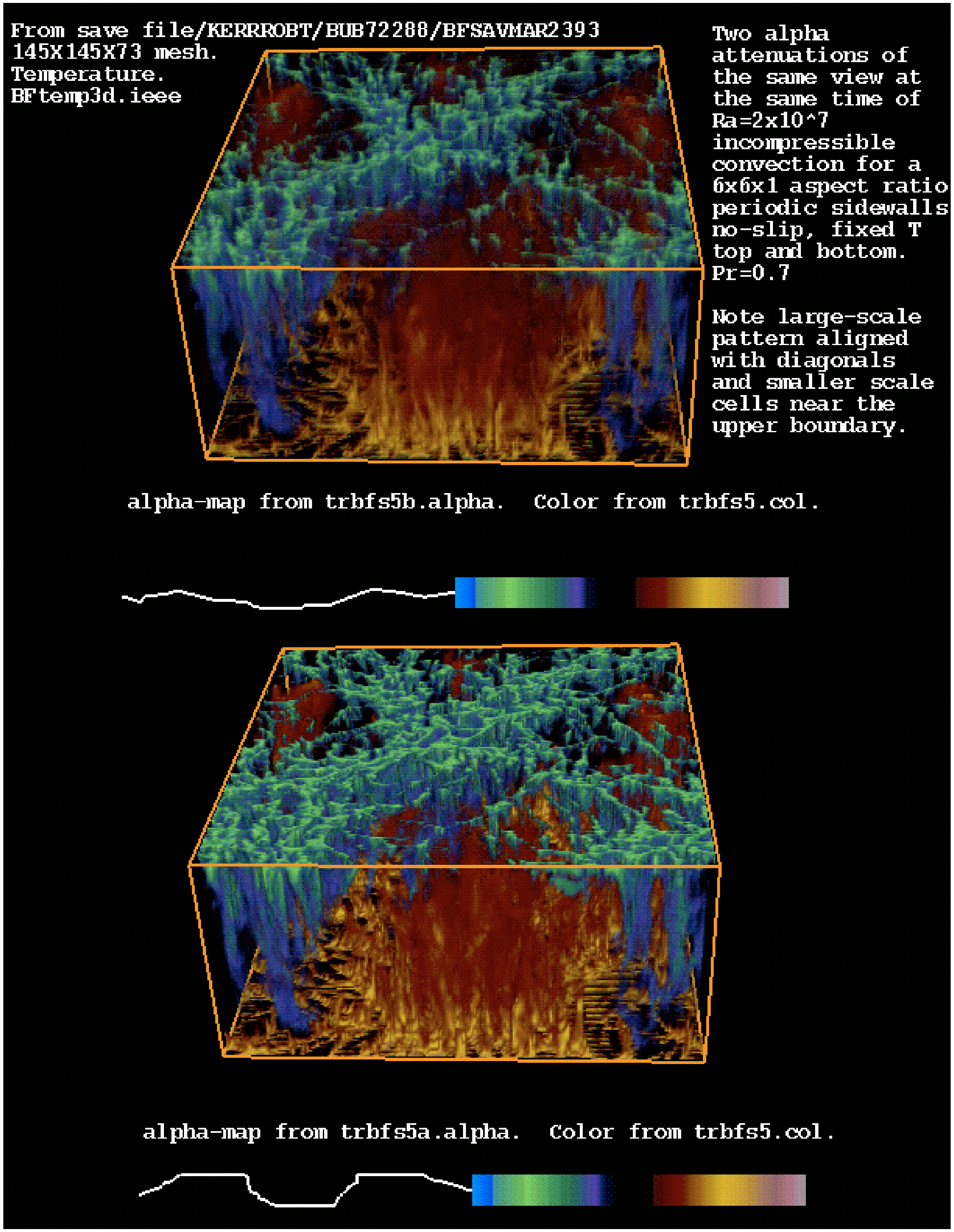
Forced rotating stratified turbulence



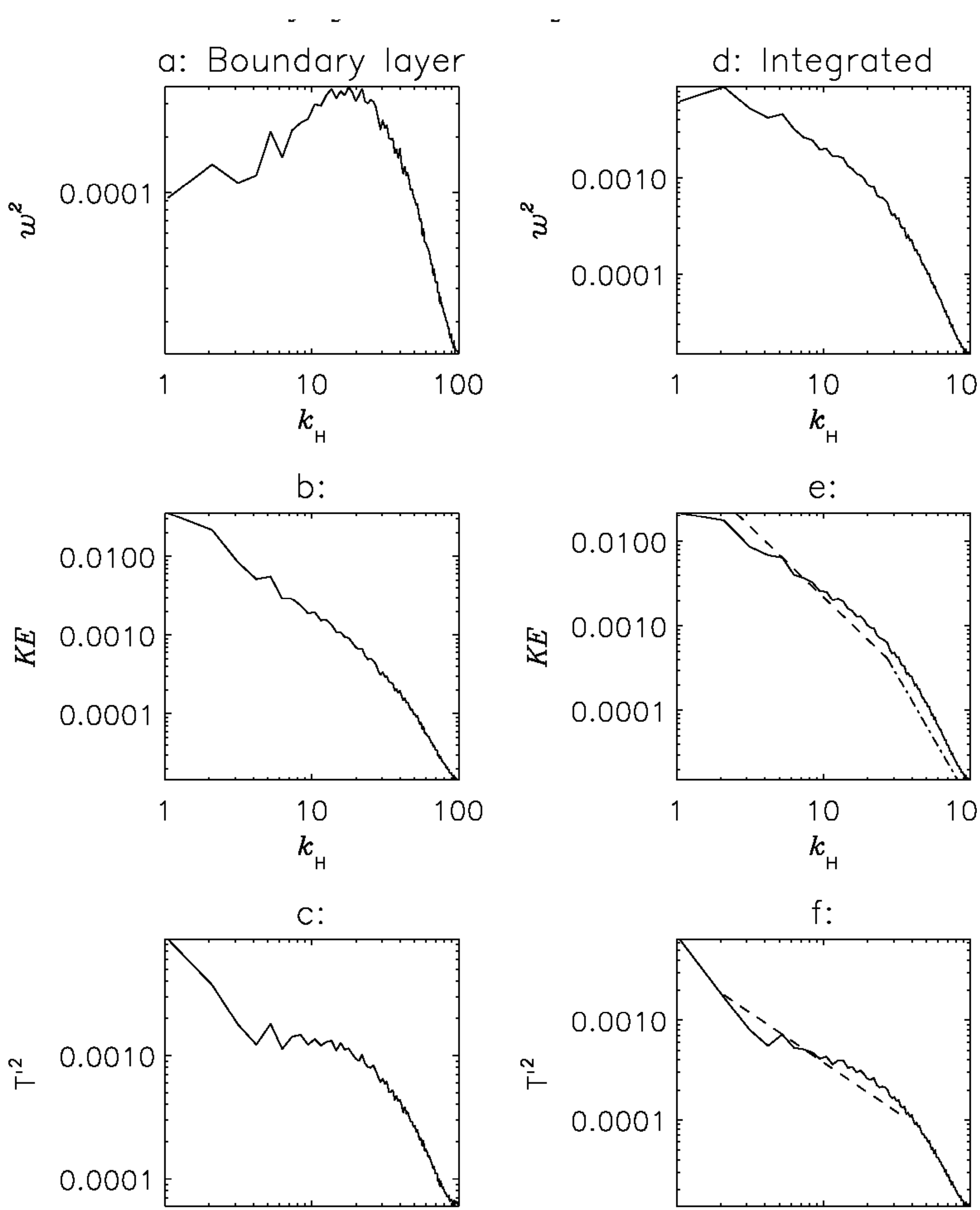

From, Smith/Waleffe, JFM (2002) p17. Two contour levels of the zonal velocity; $Fr = 0.21$, and (a) $Ro = 0.42$ $(N/f = 2)$ and $T \approx= 170$; (b) $Ro = 0.10$ $(N/f = 0.5)$ and $T \approx 100$. The dark contours represent positive zonal flow.

Below: $Fr = 0.21$, $Ro = 21$, $(N/f = 100)$ and $T = 2200$






Volume rendering of temperature for one time from the $Ra = 2 \times 10^7$ calculation. Note the dominant large-scale diagonal pattern, which persists for all Rayleigh numbers simulated and all times.




Time-averaged horizontal spectra for $Ra = 2 \times 10^7$. Spectra are collected cylindrically in planes with $k_h = (k_x^2 + k_y^2)^{1/2}$. Minimum wavenumber is $2\pi/6$, where 6 is the lateral dimension of the box. Vertical axes are in simulation coordinates and have not been normalized to be dimensionless as in figures 9 and 10. (a-c) Spectra at a plane near the upper wall, $z = 0.92$. (d-f) Integrated spectra across the box. (a,d) Vertical velocity squared. (b,e) Total kinetic energy. (c,f) Temperature variance

# **Conservation laws and scaling**

Three dimensional Navier-Stokes, inviscid conservation laws

- Kinetic energy=$\frac{1}{2}\int |u^2| d^3x = \int |u^2(k)| d^3k$
- Helicity=$\int \vec{u} \cdot (\vec{\nabla} \times \vec{u}) d^3x =$
  $Re(\int i\vec{u}(k) \cdot (\vec{k} \times \vec{u})^* d^3k)$
- Downscale cascade of energy
- $E_{11}(k) = C_1 \epsilon^{2/3} k^{-5/3}$, Struc Func.: $S_{11}(r) = C_1 \epsilon^{2/3} r^{2/3}$

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## 1 Structure functions

The $p-$th order longitudinal structure function

$$S_p^L(|r|) = \overline{(\overrightarrow{u}(\overrightarrow{x} + \overrightarrow{r}) - \overrightarrow{u}(\overrightarrow{x}))^p} \tag{3}$$

Spectrum versus 2nd order structure function, if

$$E_{11}(k) = \frac{1}{2}u_1^2(k) = C_1|\epsilon|^{2/3}k^{-5/3} \tag{4}$$

then

$$S_2^L(r) = S_2 = C_1|\epsilon|^{2/3}r^{2/3} \tag{5}$$

where $C_1$ is the one-dimensional Kolmogorov constant and $|\epsilon|$ is the energy cascade rate.

Three-dimensional forward energy cascade, Kolmogorov's 4/5 law [Kolmogorov, Frisch, 1997]:

$$S_3(r) = -\frac{4}{5}\epsilon r \tag{6}$$

That is negative and linear.

The backward two-dimensional energy cascade Lindborg [Lindborg, 1999]

$$S_3(r) = \frac{3}{2}|\epsilon|r \quad . \tag{7}$$

that is positive and linear

Cho and Lindborg figures

- Tropospheric 2nd order structure functions. Isotropic 3D relation between longitudinal and transverse 2nd order structure functions is found for $r < 500$km where $S_11 \sim r^{2/3}$ and the isotropic 2D relation is obeyed at large scales where $S_{11} \sim r^2$ is found.

- Stratospheric 2nd order structure functions. For $r < 500$km where $S_11 \sim r^{2/3}$, the longitudinal and transverse structure functions are coincident, which is neither 2D nor 3D isotropic. For large scales, the transverse structure function is much larger than even the 2D isotropic prediction.

- Stratospheric 3rd order structure function. Isotropic contributions, including longitudinal. Negative $r$ dependence for $r < 500$km indicating a forwards energy cascade even though the horizontal scale is much greater than the vertical. At large scales a postive $r^3$ dependence indicating upscale energy transfer, not downscale enstrophy transfer. This is consistent with the Smith and Waleffe numerics.

- Strospheric, anisotropic 3rd order structures indicate effects of rotation and latitude.

- Tropospheric isotropic 3rd order structure functions swith sign dependent upon latitude. Suggestive of compression and expansion of large-scale cells in the atmosphere (the Hadley circulation).

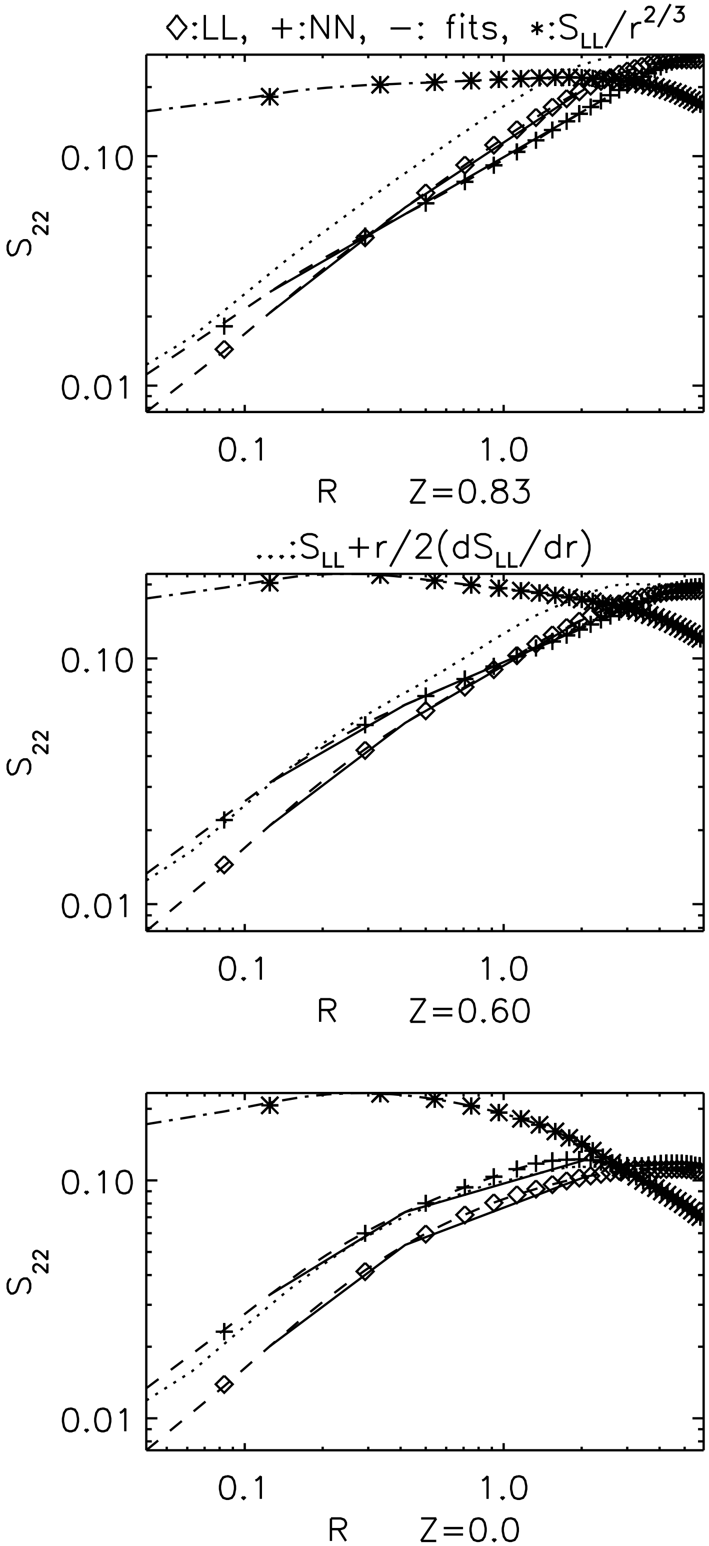



Figure 1: Comparison of 2nd order structure functions at three heights in Rayleigh-Bénard convection at $Ra = 2 \times 10^7$. Shown are $S_2^L(r)$, $S_2^T(r)$, the 3D prediction for $S_2^T(r)$ based upon $S_2^L(r)$, and $S_2^L(r)/r^{2/3}$. Only the 3D prediction for $S_2^T(r)$ is shown because the 2D prediction would be even further from the data where $S_2^T(r) \approx S_2^L(r)$. $S_2^L(r)/r^{2/3}$ is shown to demonstrate over what regimes an $r^{2/3}$ regime is found. The smallest $\Delta r = 0.04$ and only every 4th point starting t $r = 0.08$ is shown with a symbol.

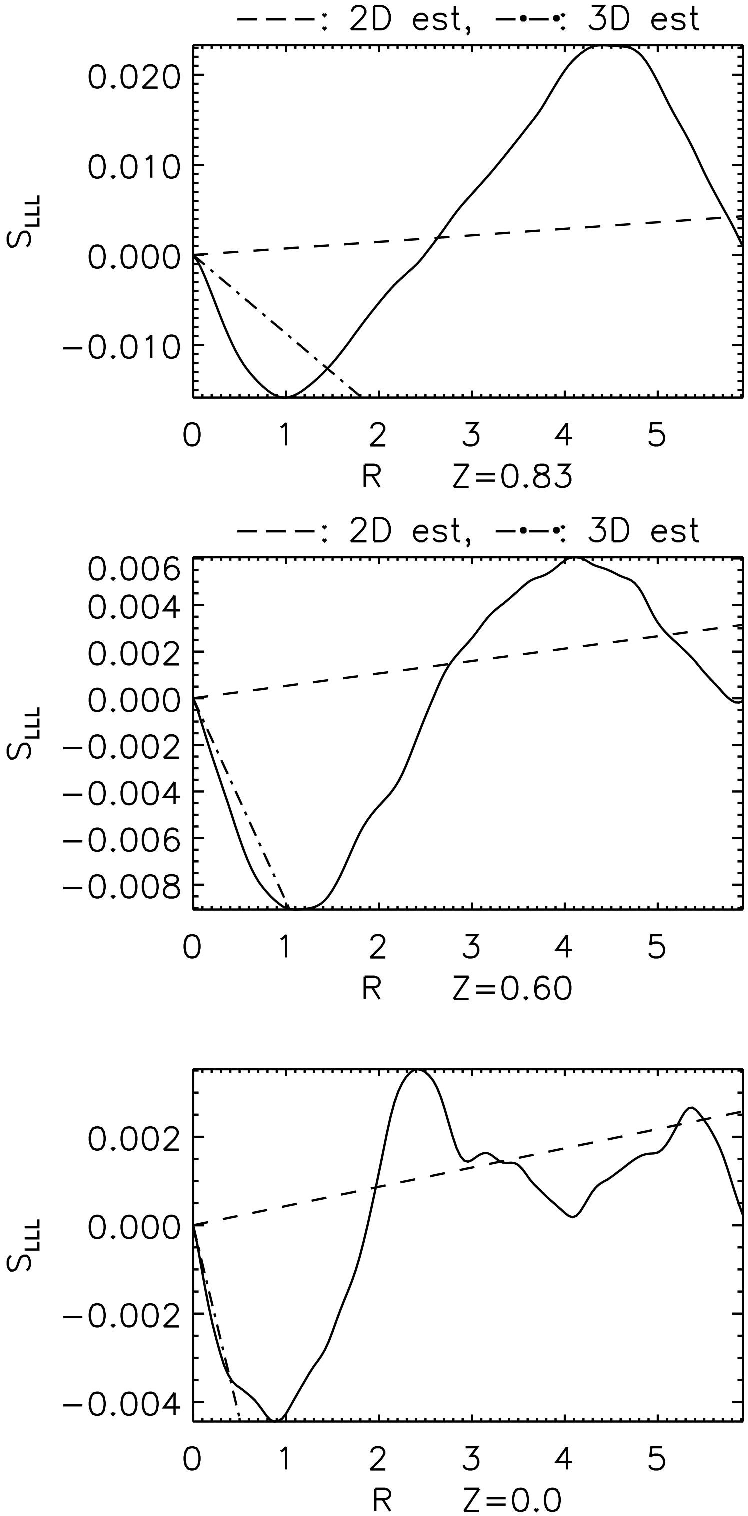



Figure 2: Comparison of 3rd order structure functions at three heights in Rayleigh-Bénard convection at $Ra = 2 \times 10^7$. Dot-dashed is the $-\frac{4}{5}\epsilon r$ 3D prediction. The dashed line is the $\frac{3}{2}|\Pi_u|r$ 2D prediction.f

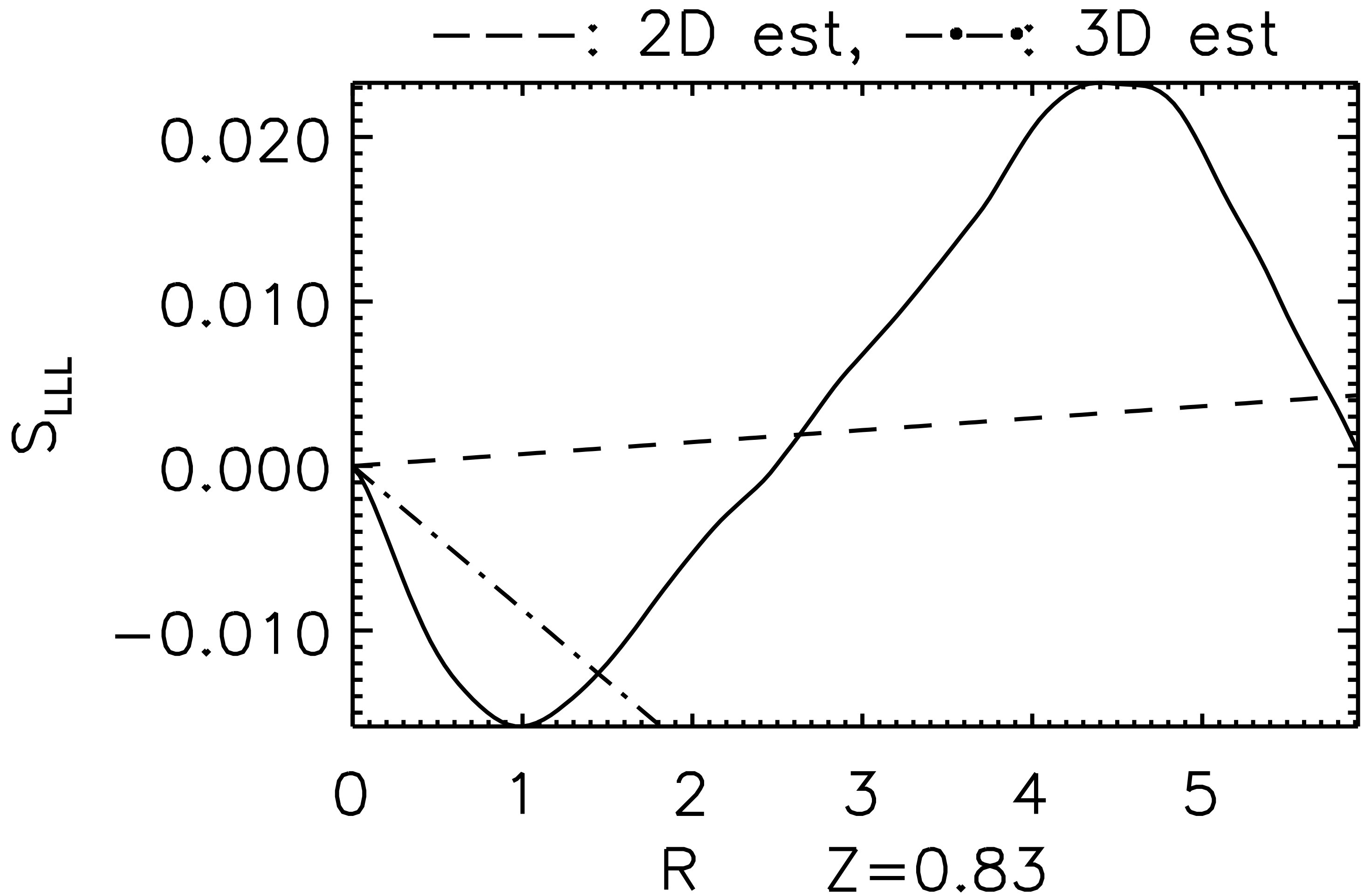
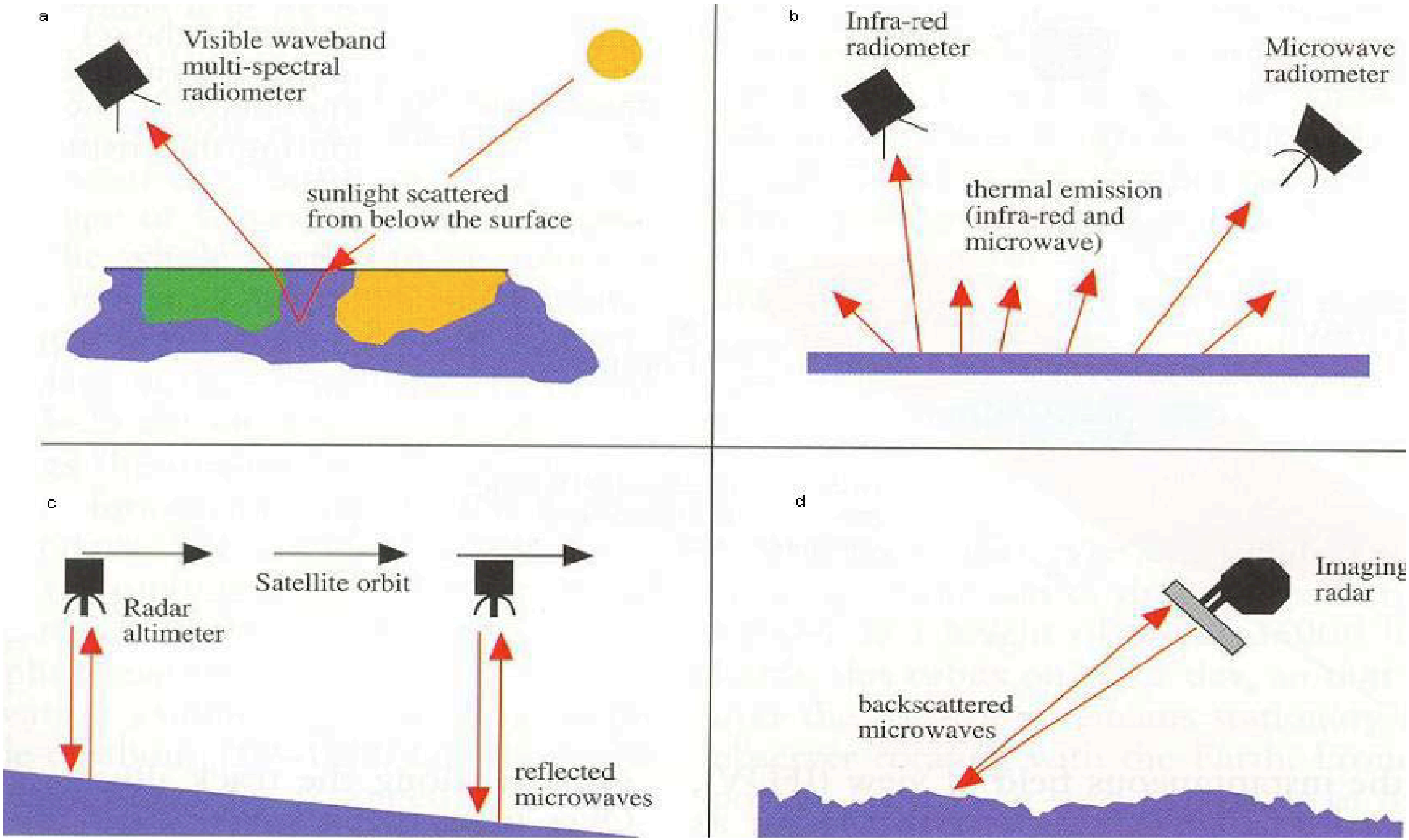
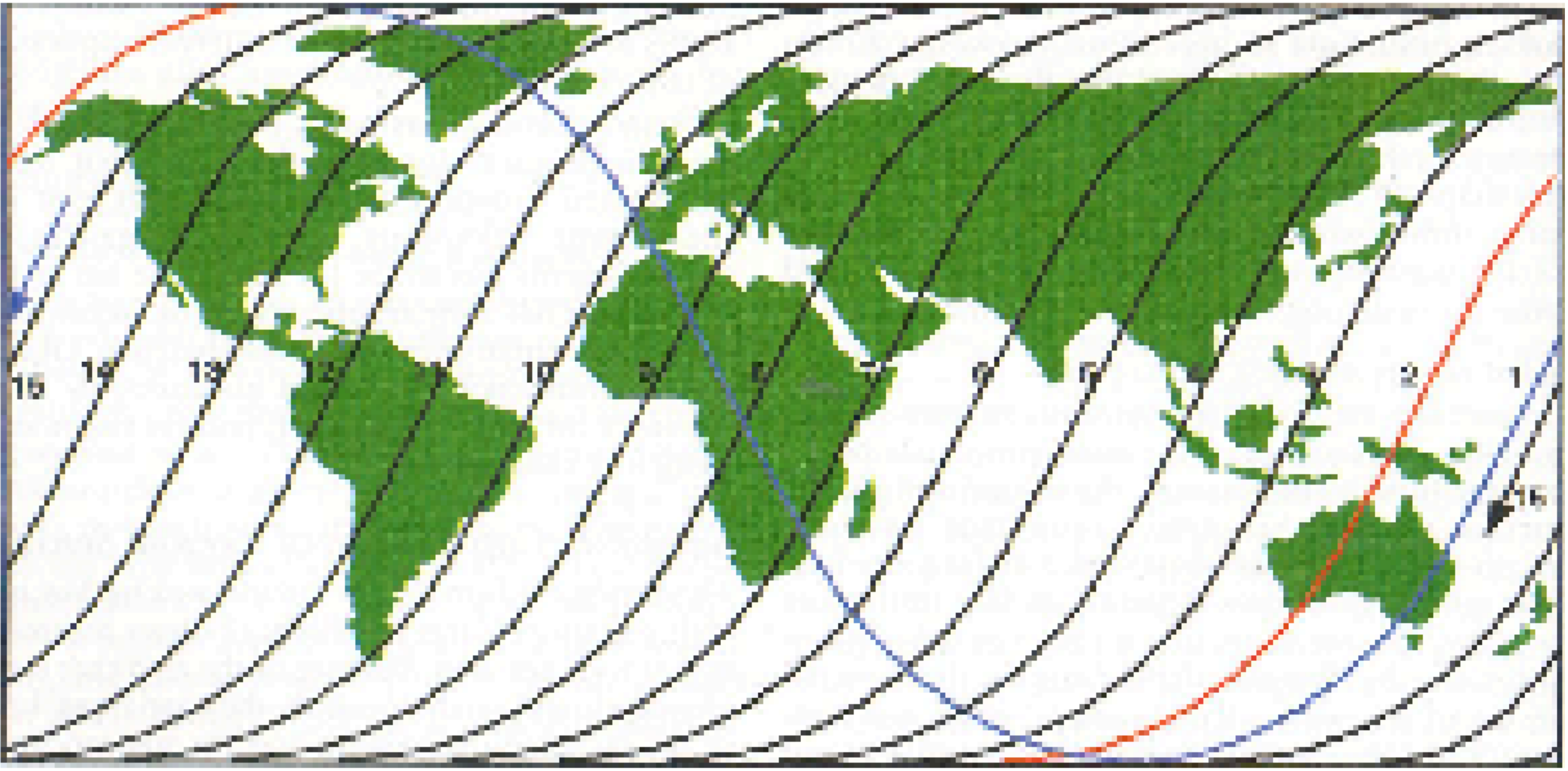


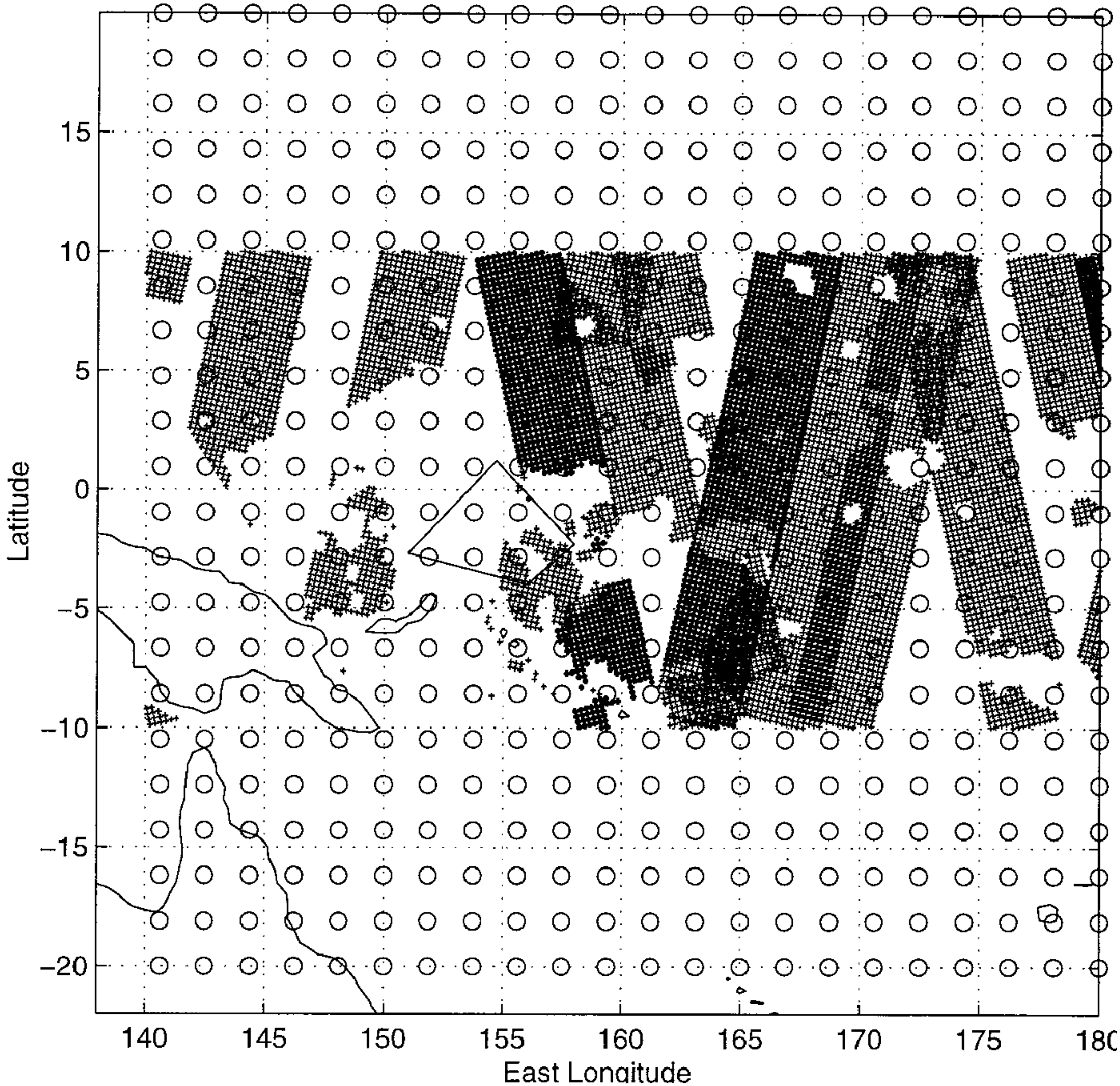

Figure 3: Third-order structure function from the top of the boundary layer in thin domain Rayleigh-Bénard convection. Function is linearly decreasing at small-scales, consistent with 3D turbulence and linearly increasing at large scales. But the slope is consistent with the upper atmosphere, not 2D turbulence. [Kerr,2001]



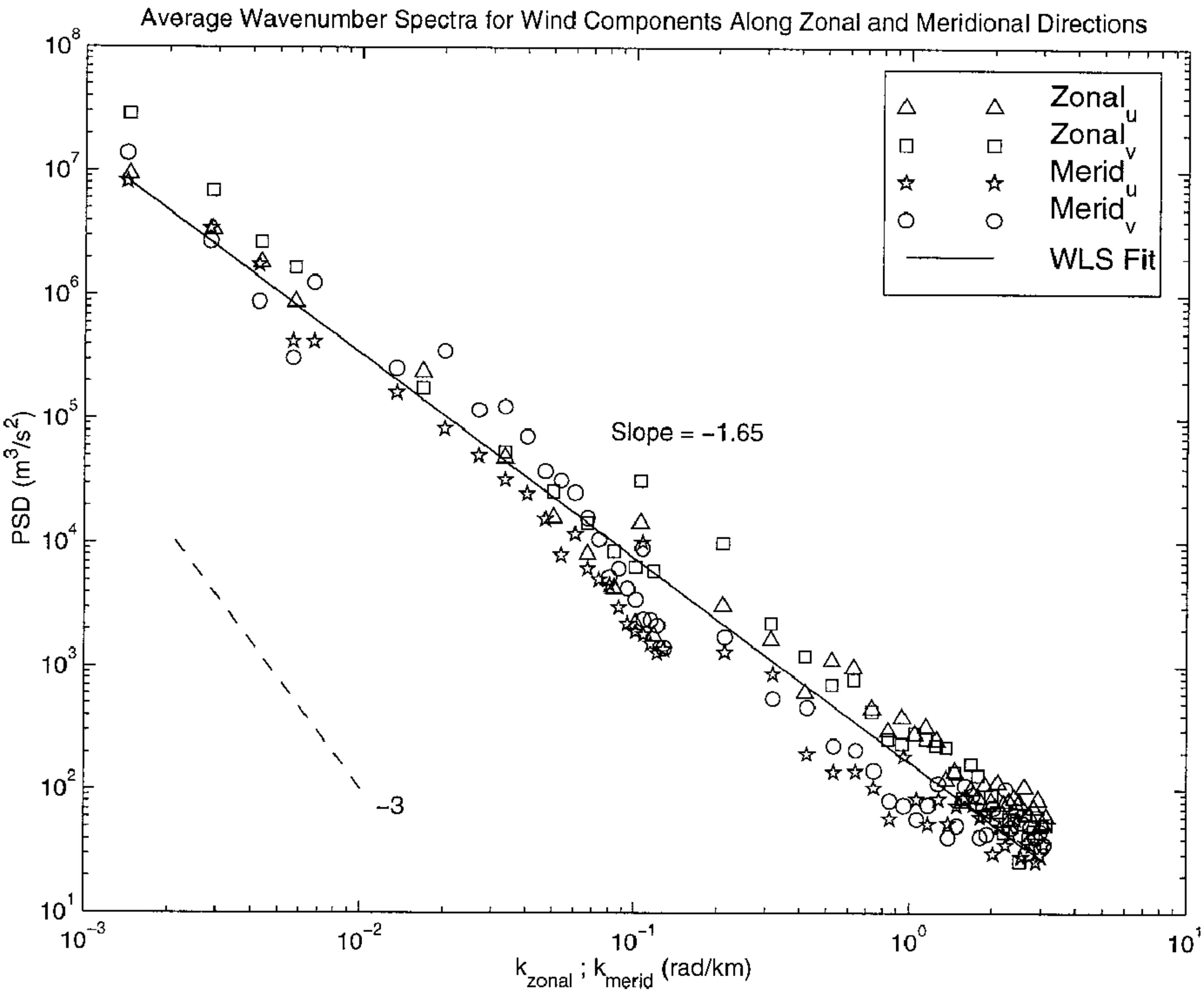

Types of satellite observations. The last one is a scatterometer.



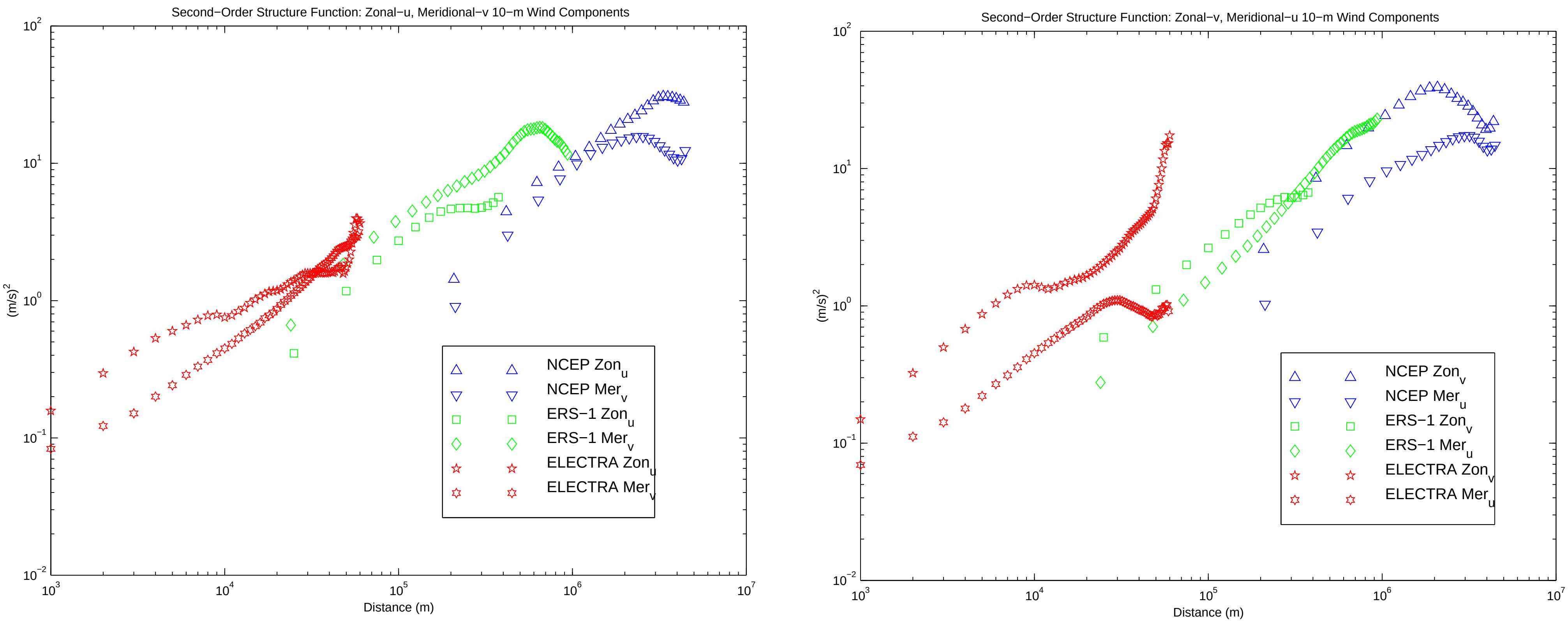

Typical coverage of a polar satellite in a day.



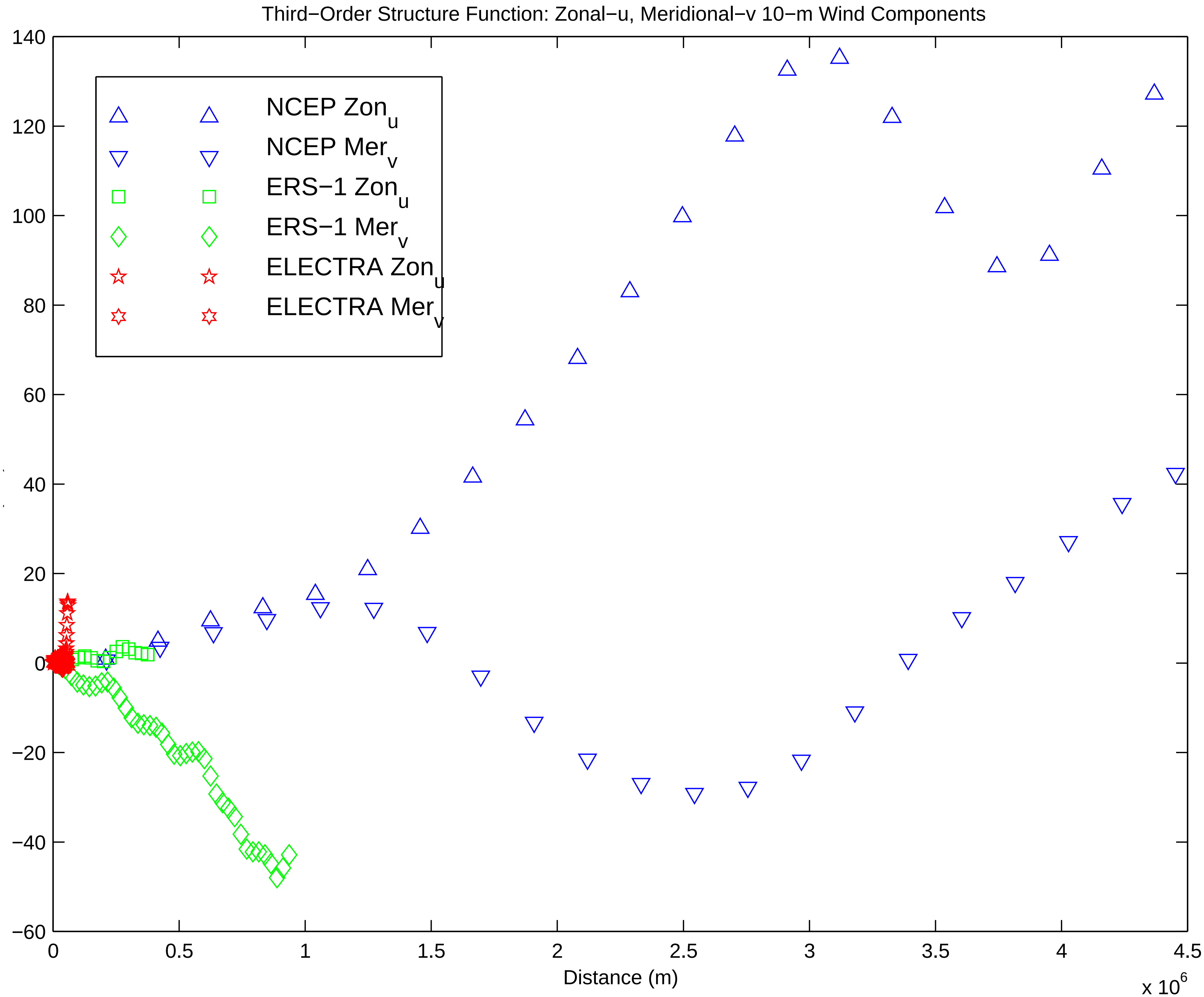

NCEP reanalysis locations ∘ and *ESR-1* scatterometer observations (hashes). Note the TOGA COARE IFA polygon (where the Electra data was taken) is outlined as well.



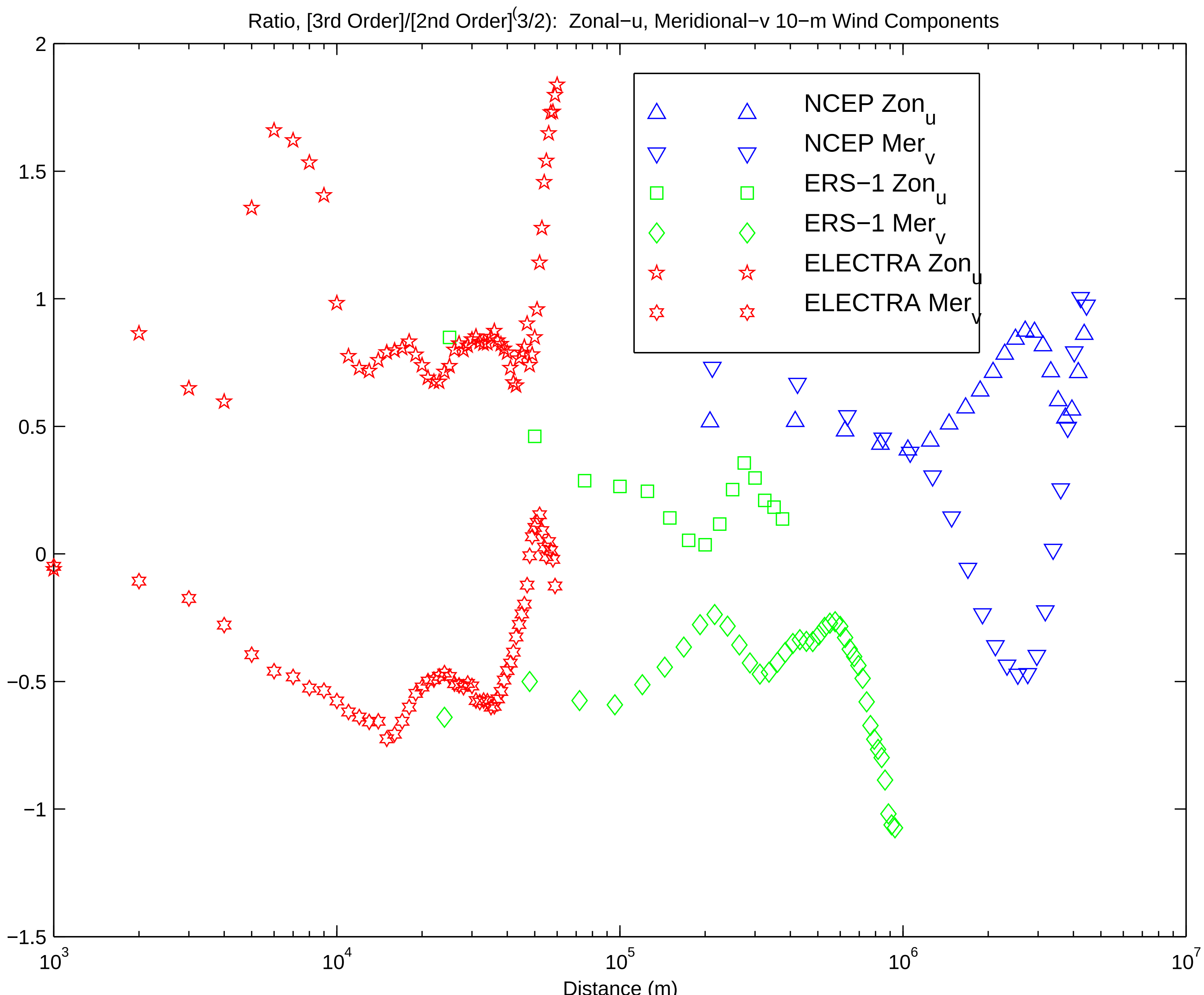

Energy spectra. Longitudinal: Zonal-y and Meridional-v. Transverse: Zonal-v and Meridional-u. All are roughly the same magnitude, which is not the expectation for either 2D or 3D isotropic turbulence.




Comparison of longitudinal and transverse 2nd order structure function. Note they are roughly the same order of magnitude.




ZUMV3a.pdf $S_{LLL}$ Lin-lin plotting. Meridional-v scatterometer is clearly negatively linear. Meridional-v NCEP reanalysis is approximately positively linear. Electra data is not seen clearly.




ZUMVrb.pdf Ratio $S_{LLL}/S_{LL}^{3/2}$ Log-lin plotting. Zonal-u is generally positive (expanding) and meridional-v is generally negative (compressing).

## Dynamically generated 3D Covariances

If the time-period is long enough, the evolved 3D covariances also depend on the dynamics:

$$\mathbf{B}_{(x(t_n))} = \mathbf{M}_{n-1} \dots \mathbf{M}_1 \mathbf{M}_0 \mathbf{B}_{(x(t_0))} \mathbf{M}_0^T \mathbf{M}_1^T \dots \mathbf{M}_{n-1}^T$$

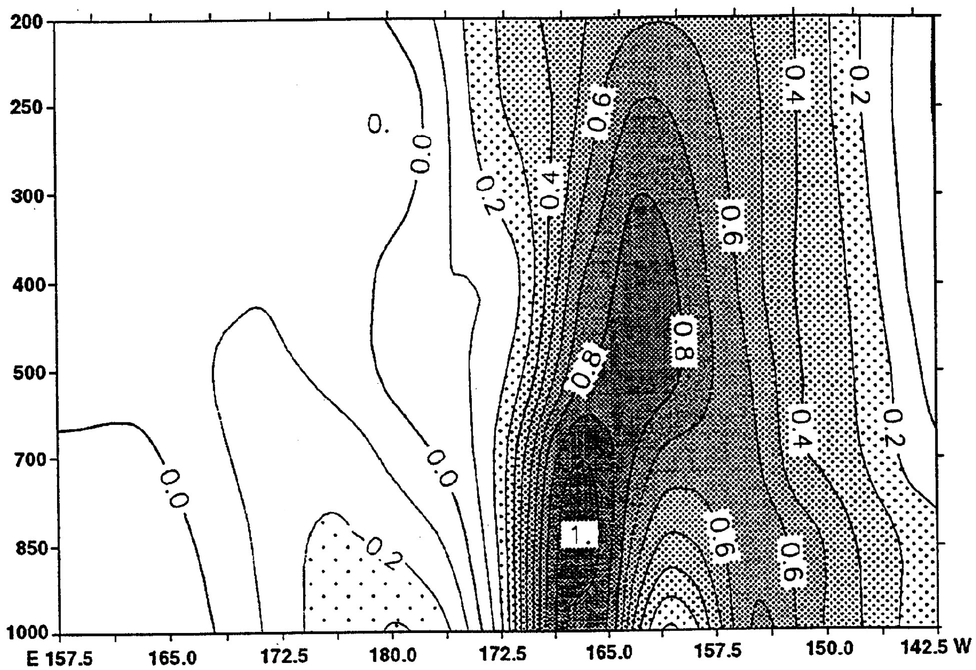
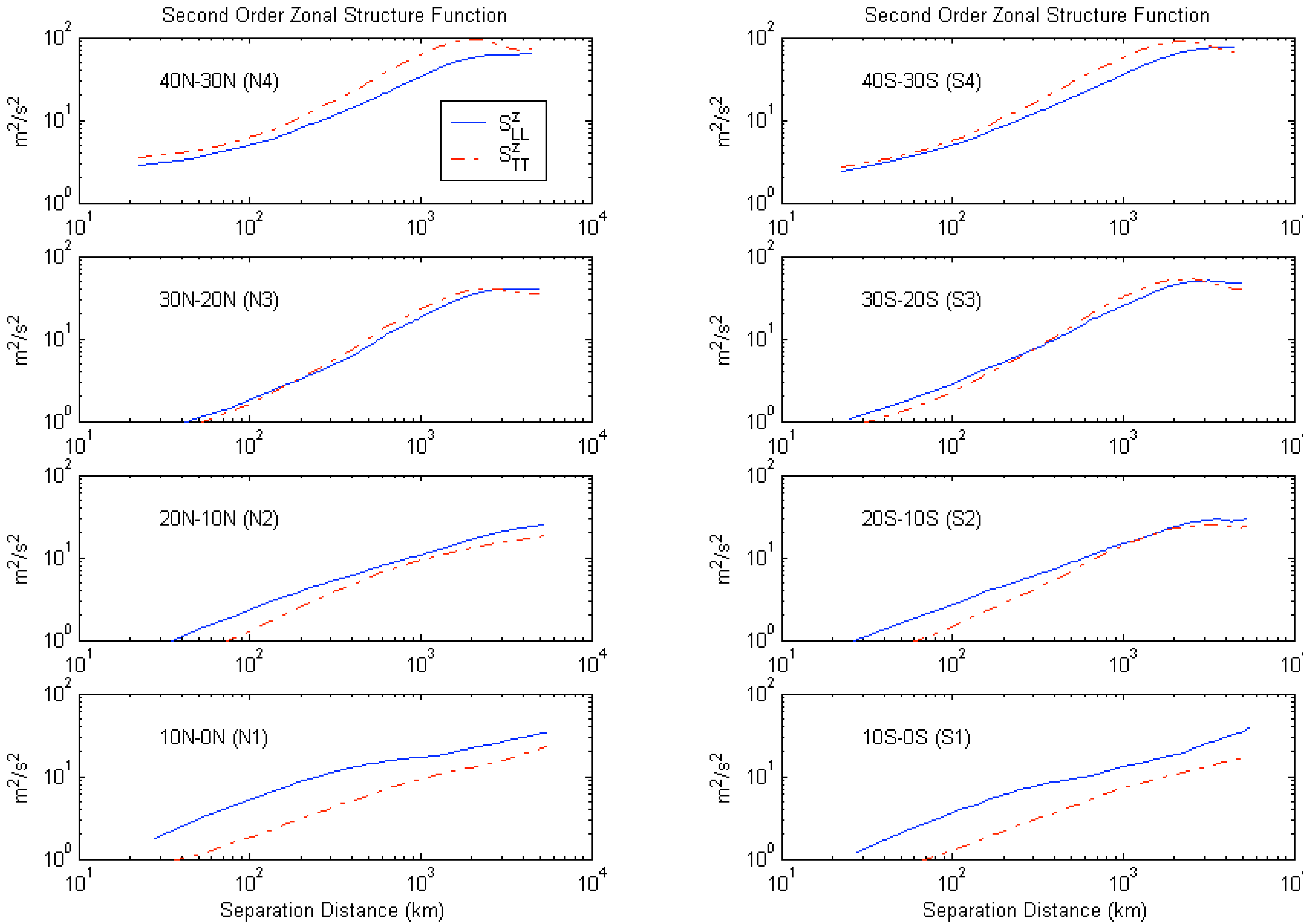



Figure 12. Cross-section of the 4D-Var structure function, obtained with an observation at location (42°N, 168.75°W, 1000hPa). Contour interval: 0.1m.

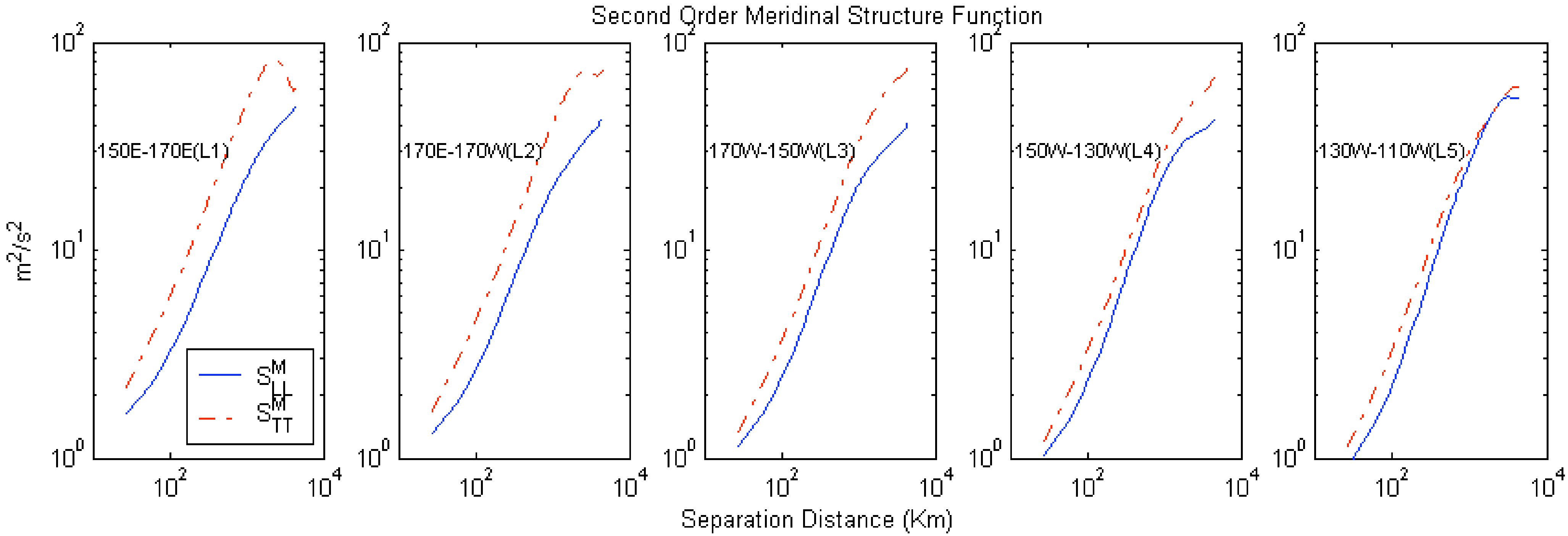
Thépaut, Jean-Nöel, P. Courtier, G. Belaud and G Lemaître: 1996 "Dynamical structure functions in a four-dimensional variational assimilation: A case study" *Quart. J. Roy. Met. Soc.*, **122**, 535-561

| Notation | | | |
|---|---|---|---|
| | $< (u(x+r) - u(x))^2 >$ | 2nd zon long | |
| | $< (v(x+r) - v(x))^2 >$ | 2nd zon transverse | |
| | $< (v(y+r) - v(y))^2 >$ | 2nd mer long | |
| | $< (u(y+r) - u(y))^2 >$ | 2nd mer transverse | |
| | $< (u(x+r) - u(x))^3 >$ | 3rd zon long | diagonal |
| | $< (u(x+r) - u(x))(v(x+r) - v(x))^2 >$ | | diagonal |
| | $< (v(x+r) - v(x))^3 >$ | 3rd zon transverse | off-diagonal |
| | $< (u(x+r) - u(x))^2(v(x+r) - v(x)) >$ | | off-diagonal |
| | $< (v(y+r) - v(y))^3 >$ | 3rd mer long | diagonal |
| | $< (v(y+r) - v(y))(u(x+r) - u(x))^2 >$ | | diagonal |
| | $< (u(y+r) - u(y))^3 >$ | 3rd mer transverse | off-diagonal |
| | $< (v(y+r) - v(y))^2(u(y+r) - u(y)) >$ | | off-diagonal |

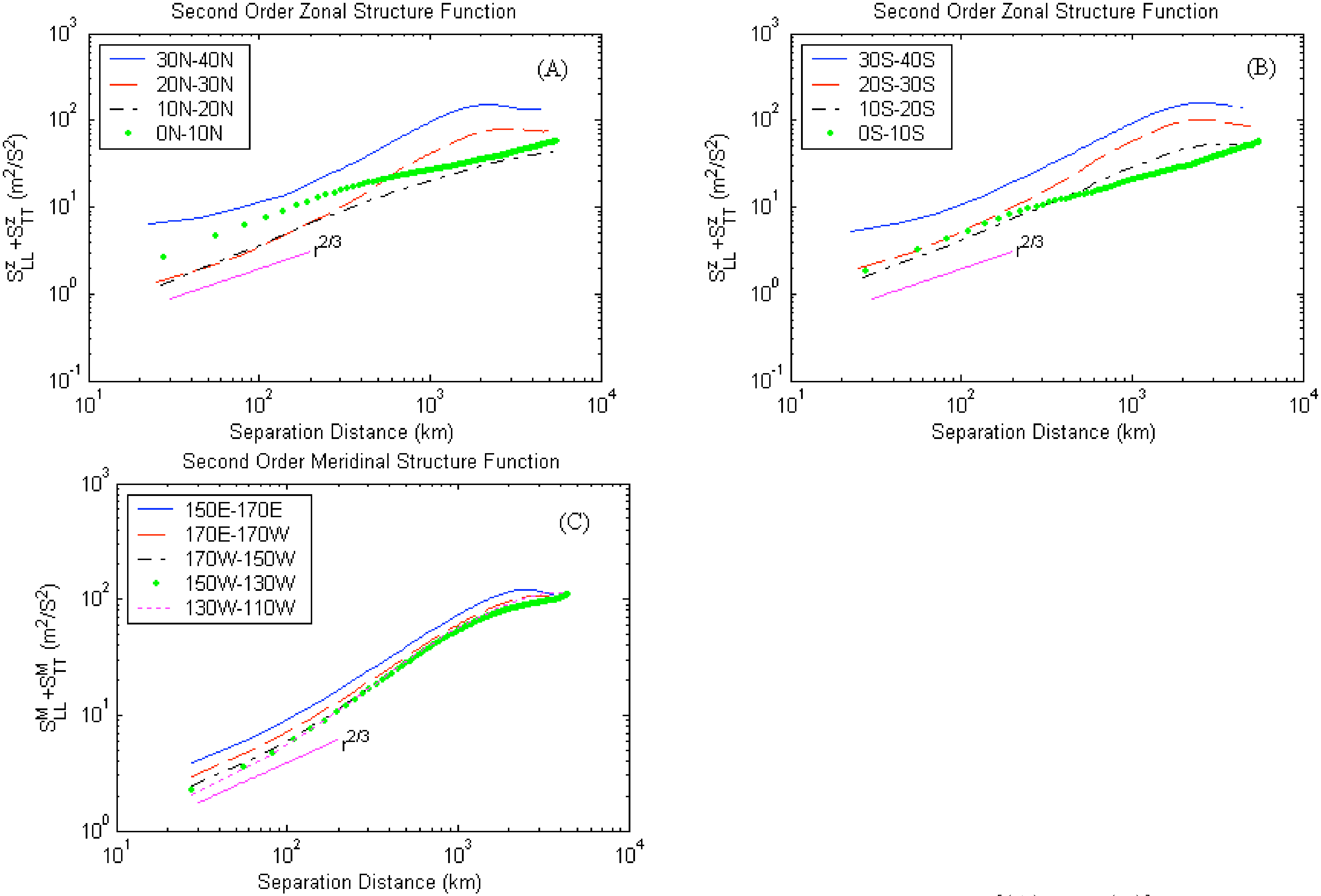
**Table 1.** Definitions of structure functions



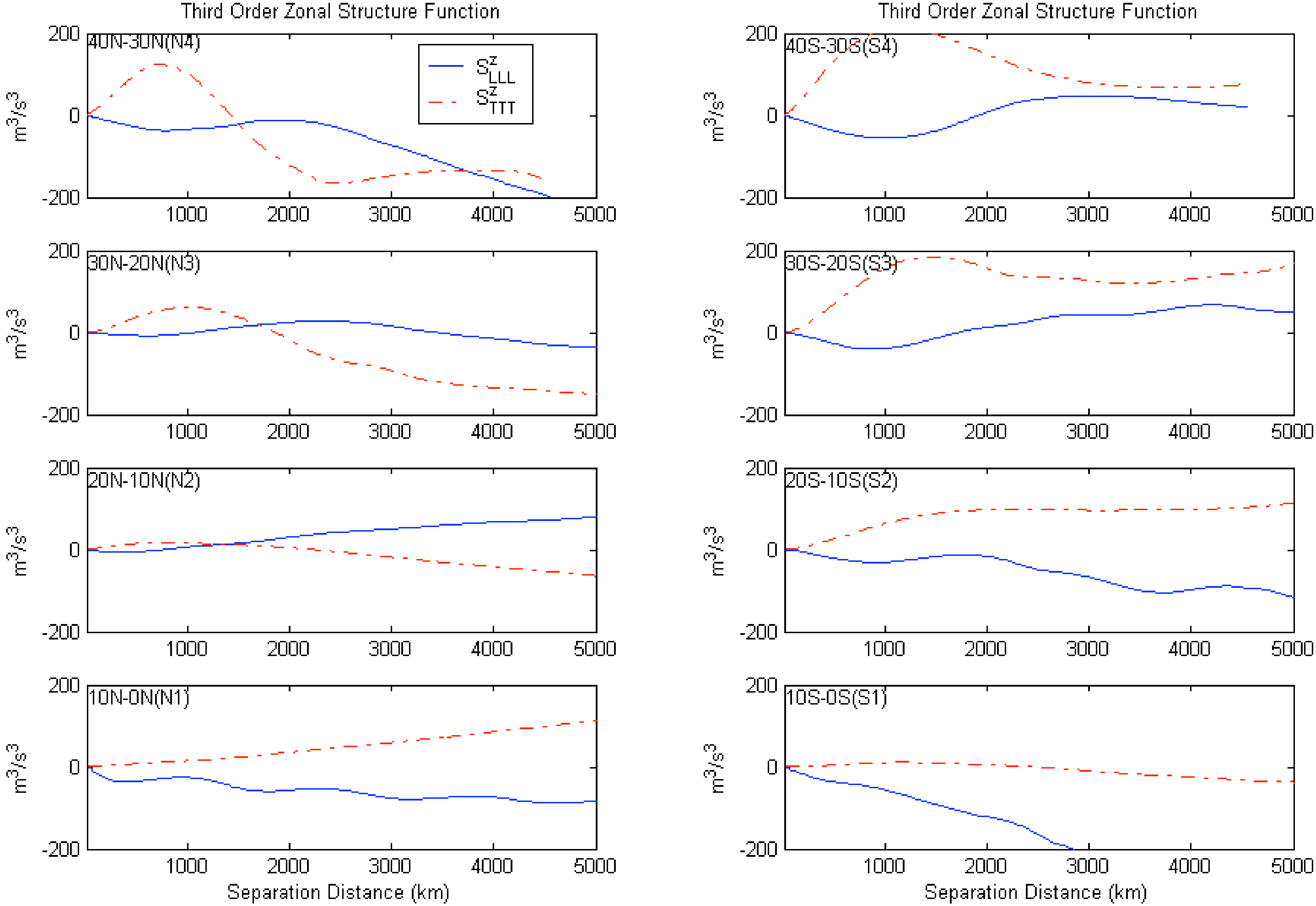

The average second order zonal structure function for eight different latitude bands in five years interval from Jan 2000 through Dec 2004. The transverse structure functions ($S^Z_{TT}$) are larger than longitudinal ones ($S^Z_{LL}$) in N4 and S4 for most of the separation distances. The phenomenon changes as they close to equator. Basically, the behavior between the longitudinal and transverse structure function is symmetric to equator.



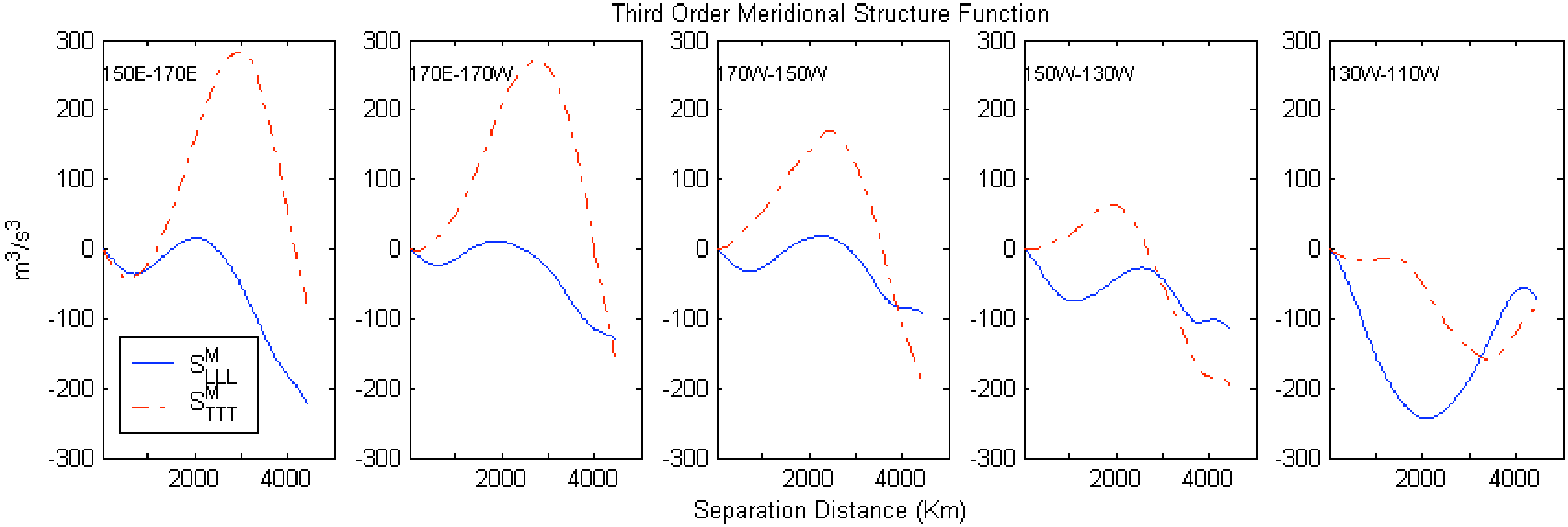

The average Second Order Meridional Structure Function for five different longitude bands in five years interval from Jan 2000 through Dec 2004. The transverse structure functions ($S^M_{TT}$) are larger than longitudinal ones ($S^M_{LL}$) for all of the longitude bands but a small part of band L5.



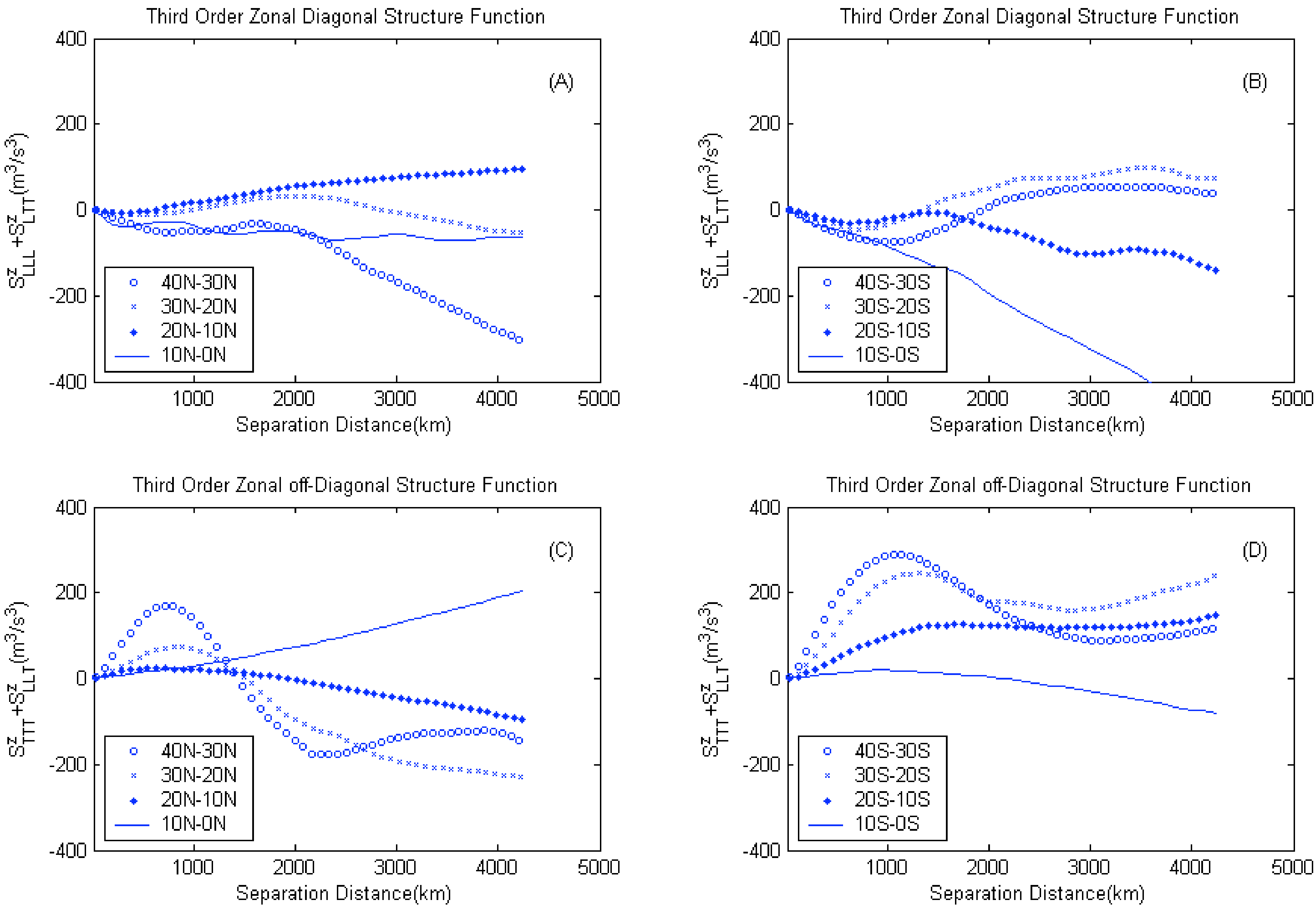

The comparisons of the second order structure function for different latitude [(A) and (B)] and longitude bands. The $r^{2/3}$ line indicates the two- third law of Kolmogorov (1941). (A) and (B) show the magnitudes of zonal structure functions decrease monotonically forwarding to the equator in the large scale of the separation distances but the band of 0N-10N. (C) shows the magnitudes of the meridional structure functions decrease monotonically forwarding to east Pacific Ocean.



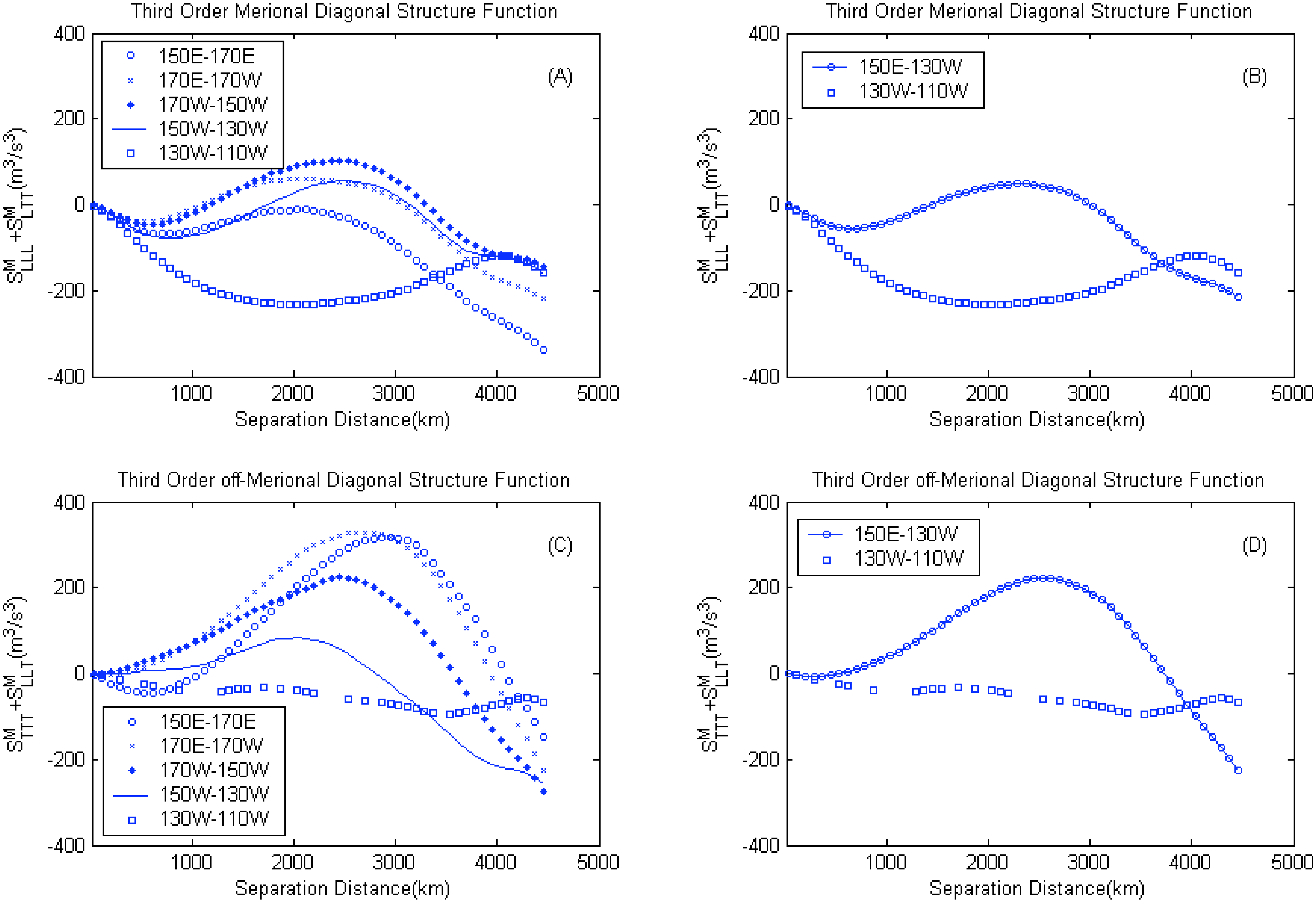

The average third order zonal structure function for eight different latitude bands in five years interval from Jan 2000 through Dec 2004. The amplitudes of longitudinal structure functions for these bands are not large, but the tendency is to decrease in the small separation distance and increase in larger separation distance. The transverse structure functions are positive in most of the bands.



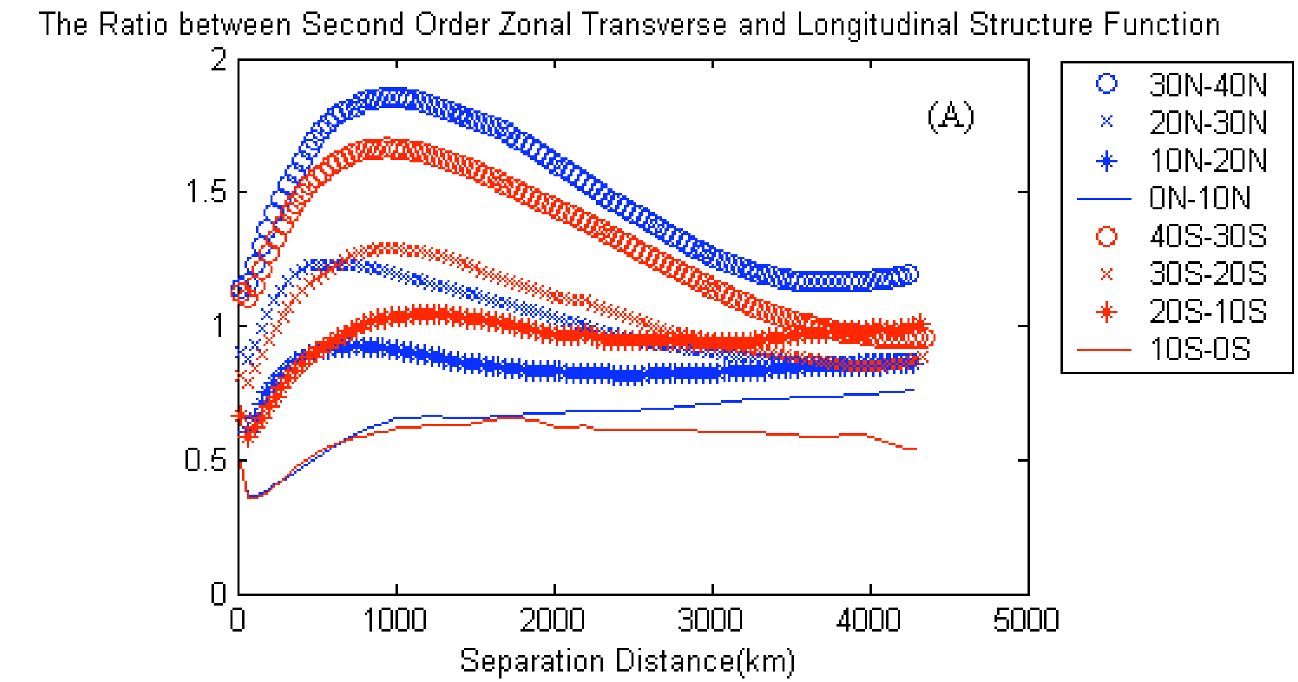

The average third order meridional structure function for five different longitude bands in five years interval from Jan 2000 through Dec 2004. The tendency of the longitudinal structure function is to decrease in smaller separation distance and increase in larger separation distance. The tendency of the transverse structure function is similar to the longitudinal one and the amplitude of them decrease forwarding to the east of Pacific Ocean.

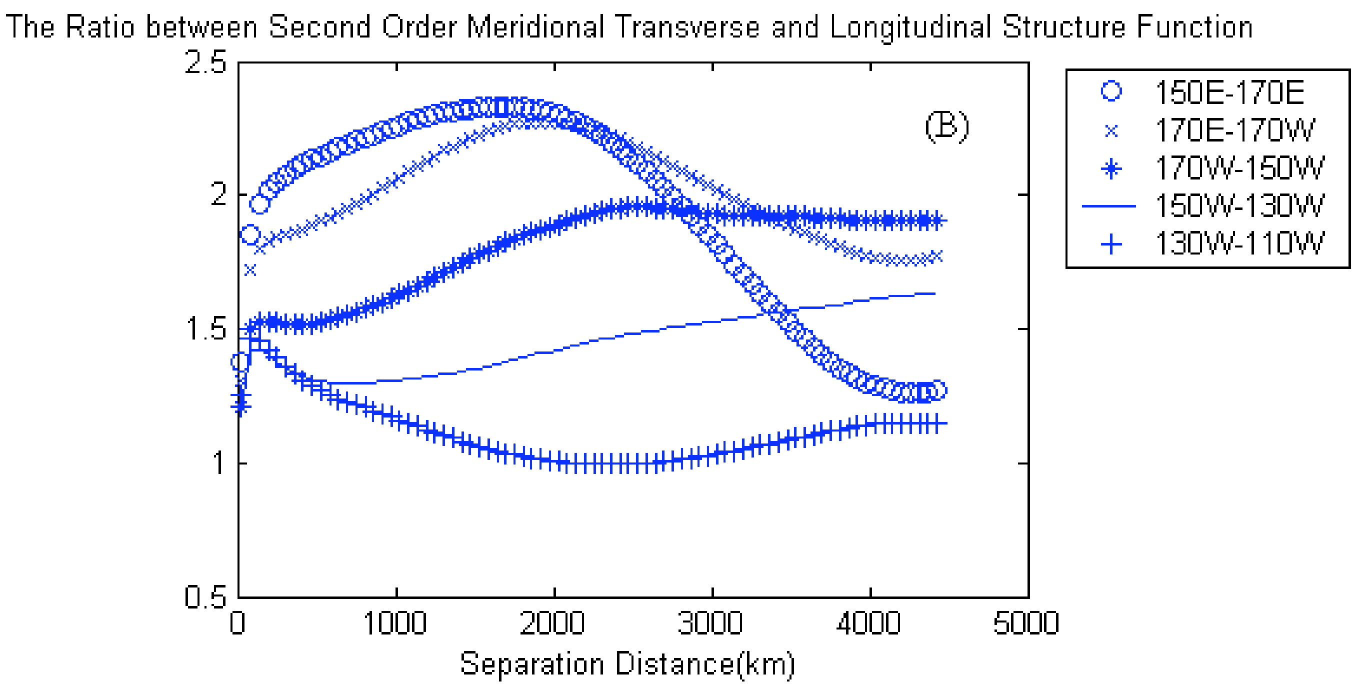



Third order zonal diagonal (A&B) and off-diagonal (C&D) structure function for eight different latitude bands in five years interval from Jan 2000 through Dec 2004. Some bands of figure (A&B) show the tendency of the positive linear behavior. The tendency of the off-diagonal (C&D) structure functions is obviously clear than diagonal ones and the magnitude decrease forwarding to the equator in smaller separation distance.




Third order meridional diagonal (A&B) and off-diagonal (C&D) structure function for five different longitude bands in five years interval from Jan 2000 through Dec 2004. The figure (B) and (D) are the combination results of the similar pattern of figure (A) and (C). The magnitudes of the diagonal and off-diagonal terms in the band 150E-130W are negative in smaller separation distance (¡1000km) and have positive linear zone (1000 2500km).




The ratio between transverse and longitudinal structure functions for zonal (A) and meridional (B) direction. The magnitudes of the ratio along the zonal direction in figure (A) decrease forwarding to the equator for all of the separation distances. The ratio is decreasing in small separation distance (∼200km) and increase to the maximum value and the tendency of the ratio is symmetric to the equator. The magnitude of the meridional ratio is larger than 1 for all of the longitude bands.

## References

[Frisch, 1997] U. Frisch 1995 *Turbulence: The legacy of A.N. Kolmogorov.*. Cambridge Univ. Press, 296 pp..

[Kolmogorov, 1941b] A. Kolmogorov 1991 The Royal Society, London. In (ed. ), pp. 434-Turbulence and stochastic processes: Kolmogorov's ideas 50 years on. Proc. Roy. Soc. London Ser. A. .

[Lindborg, 1999] E. Lindborg 1999 Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence?. *J. Fluid Mech.* **388**, 259, (1999).