Wind-wave interactions

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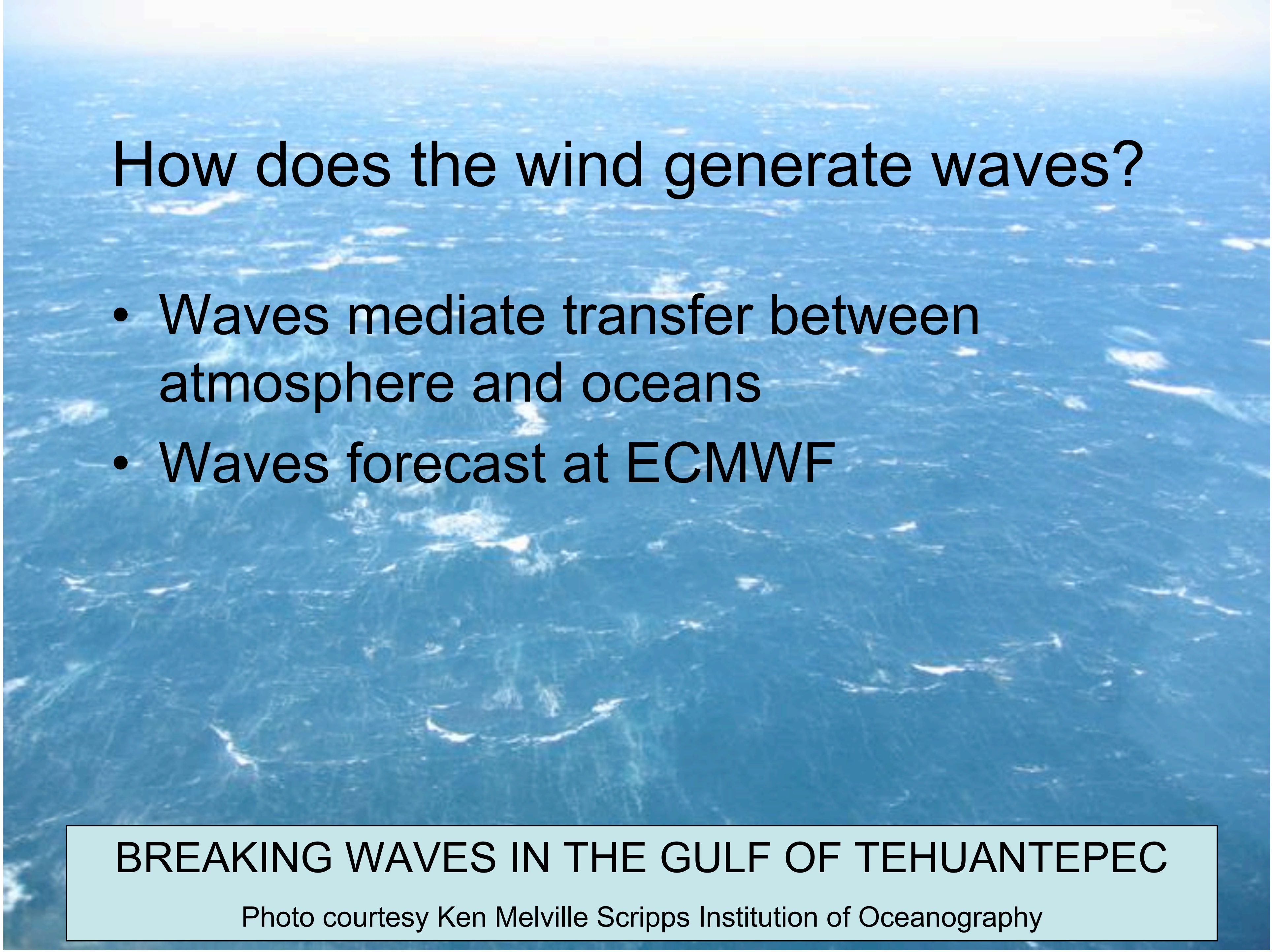
Inspiration from Michael McIntyre

How does the wind generate waves?

- Waves mediate transfer between atmosphere and oceans
- Waves forecast at ECMWF



BREAKING WAVES IN THE GULF OF TEHUANTEPEC

Photo courtesy Ken Melville Scripps Institution of Oceanography

Previous theory

$$\frac{\partial w}{\partial t} + (U - c)\frac{\partial^2 w}{\partial x^2} + (U - c)\frac{\partial^2 w}{\partial x^2} - U''w$$
$$= \text{nonlinear terms} + \text{turb stress}$$

- Critical layer (Miles)
  - – Linear inviscid normal-mode instability
- Non-separated sheltering (Belcher & Hunt)
  - – Linear instability with turbulent stress

Current work

- Previous models are
  - Linear
  - Normal mode instability for sinusoidal waves
- Questions:
  - Are nonlinear effects important?
  - Effects of unsteadiness:
    - In forcing?
    - Inherent in fluid dynamics?
- Motivated by large scale Rossby waves in atmosphere
  - (Warn & Warn; Stewartson; Killworth & McIntyre; Haynes)

Toy problem

- Channel moving at wave speed
- Initial velocity profile
- Steady forcing at lower boundary

  $w(0) = akc\cos kx$
- 2D Navier-Stokes dynamics

$$\frac{D\omega}{Dt} \approx 0$$

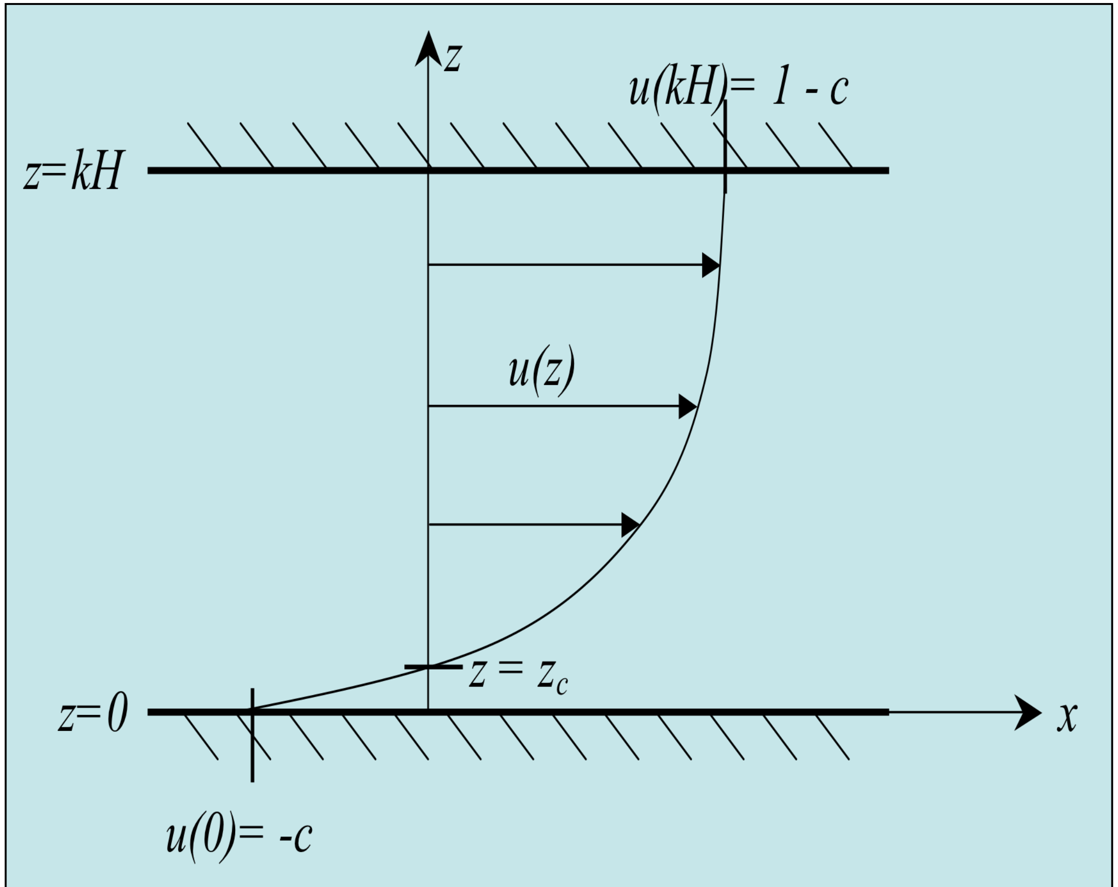
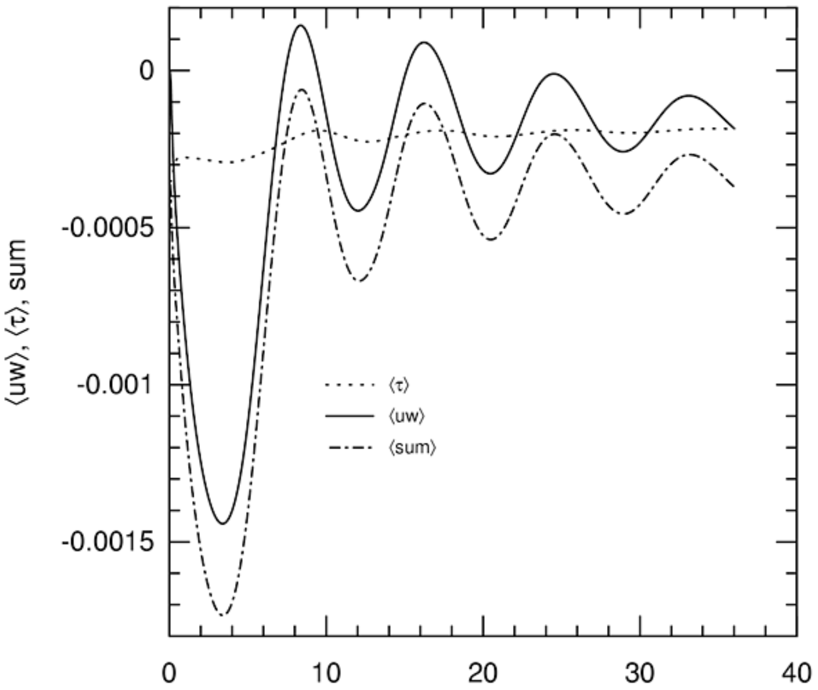


Figure 1. Sketch of the flow geometry

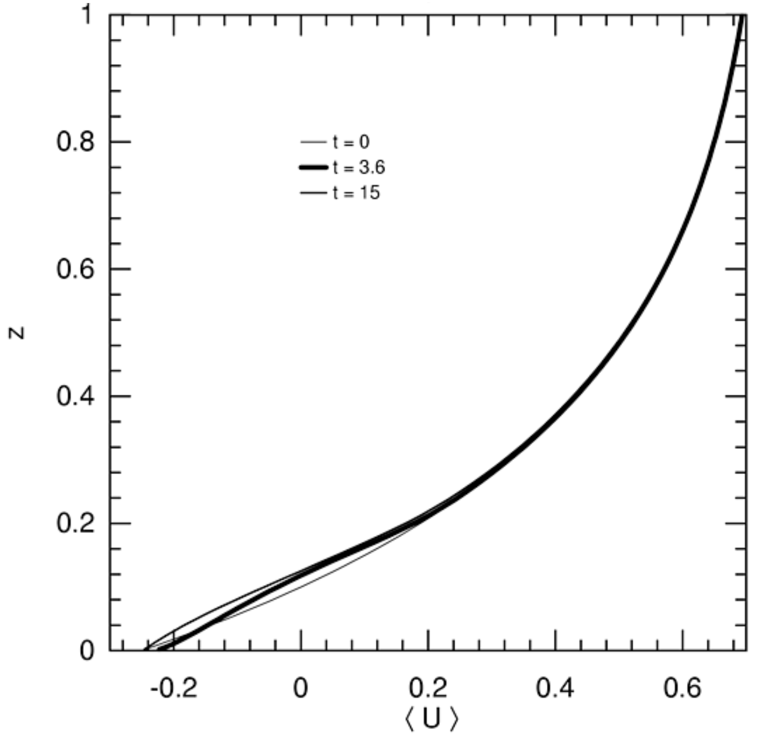
Wave induced stress at *z = 0*

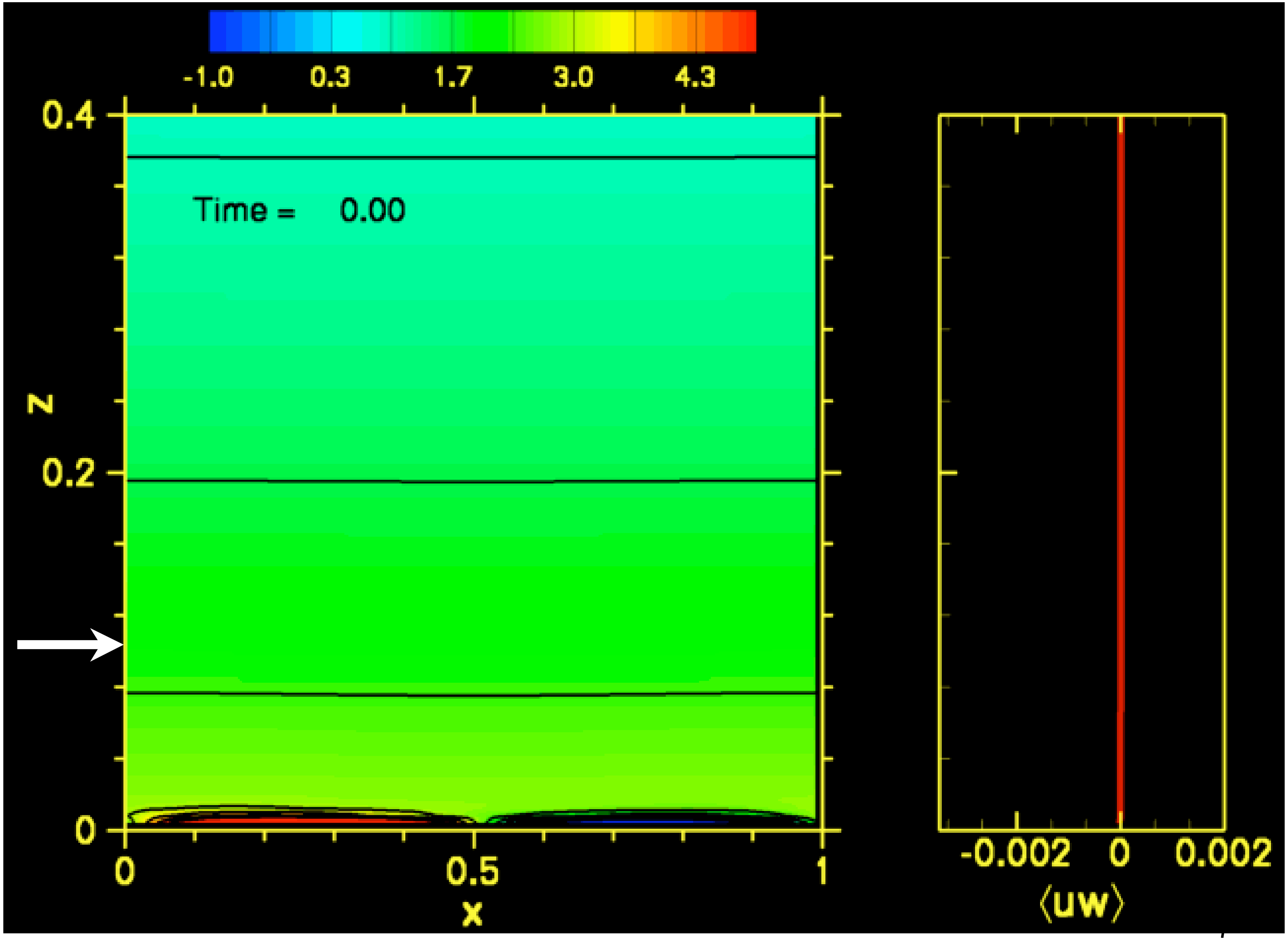
- Momentum flux from wind to waves
- Forces wave growth
- Transient!



Evolution of wind profile

- Low level wind speed reduced
- Curvature tends to zero
- Vorticity mixed to constant value across critical layer




-1.0
0.3
1.7
3.0
4.3
Time = 0.00
0.4
0.2
0
z
0
0.5
1
x
-0.002
0
0.002
⟨uw⟩

Stages in evolution

- Linear: $t = O(1)$
  - Singularity in Rayleigh equation resolved by tendency term
  - Wave growth by wave induced stress

$$\frac{\partial w}{\partial t} \approx U'' w$$

- Nonlinear wrap up: $t = O(1/\epsilon^{\frac{1}{2}})$
  - Vorticity mixed in critical layer
  - Tends to constant vorticity across critical layer

$$\epsilon = \frac{ak\,c}{kL\,U_0}$$

$$|z - z_c| \sim \epsilon^{\frac{1}{2}} L$$

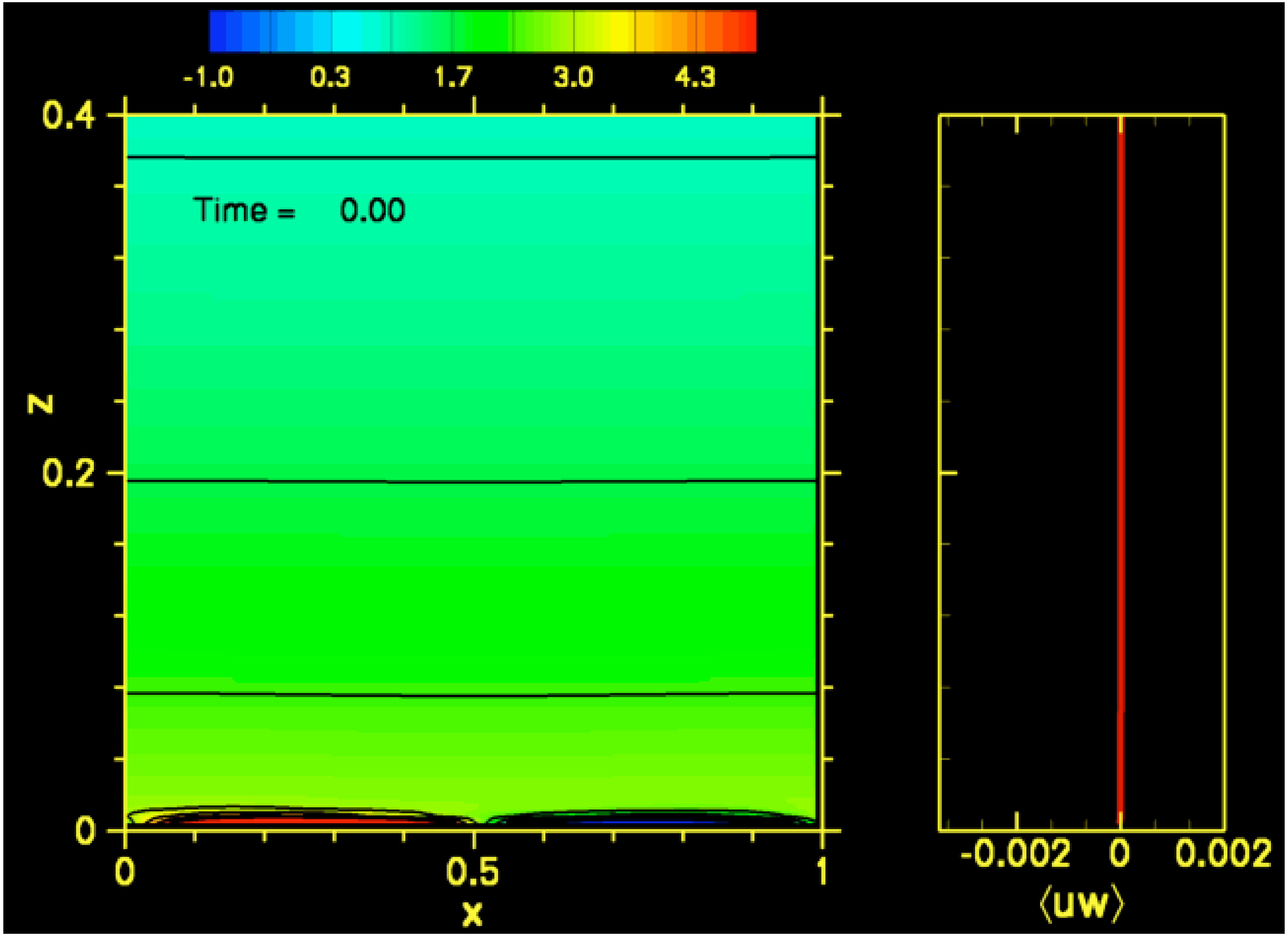
- Instability: $t \approx 3/\epsilon^{\frac{1}{2}}$
  - Flow within critical layer unstable
  - Rapid mixing of vorticity follows
  - Momentum transfer to waves arrested

Suggested role of 3d turbulence

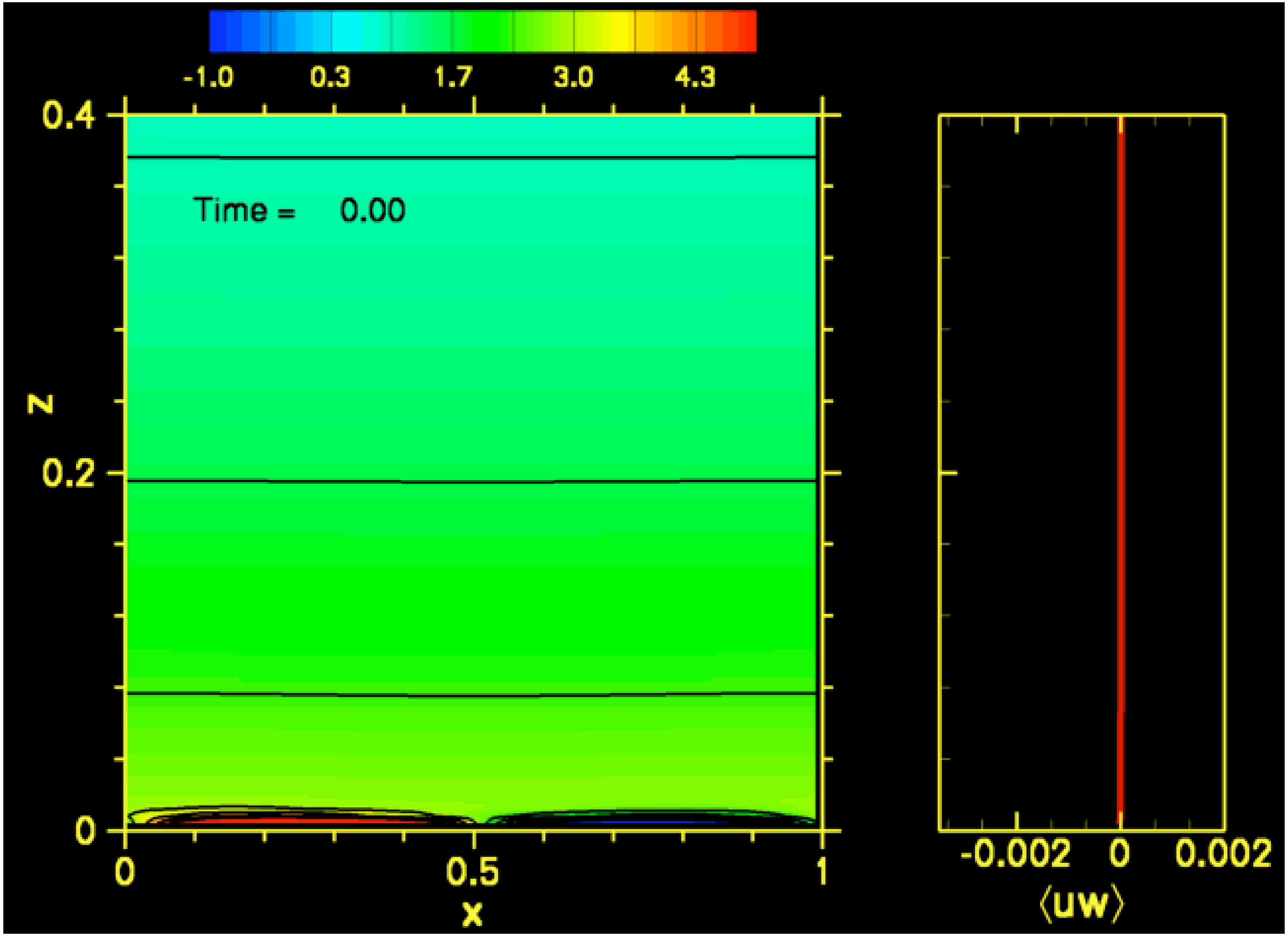
- Critical layer mechanism removes curvature
  - Timescale: transit time around critical layer: T_c
- Turbulence restores mean velocity curvature
  - Log profile for boundary layer flow
  - Timescale: eddy turnover timescale: T_t
- Toy model:
  - Relax to initial velocity profile over timescale T_t:

$$-\frac{< U > - U_0}{T_t}$$

Fast turbulence: $T_t = 0.1 < T_c = 3$

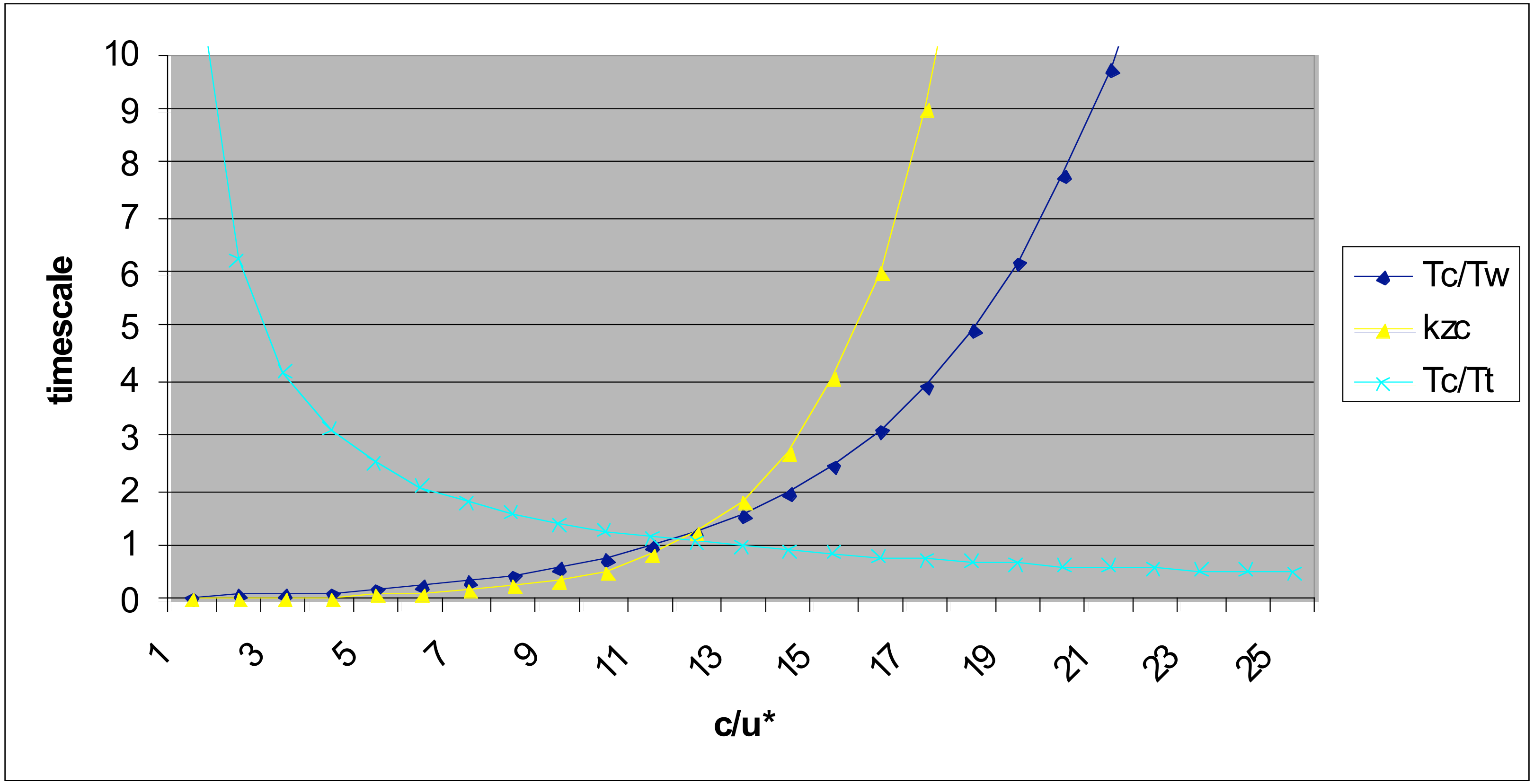


Slow turbulence: $T_t = 10 > T_c = 3$



Estimates for log layer

- Turbulent mixing: $T_t = \frac{\Delta}{u_*}$
- Critical layer turnover: $T_c = \frac{1}{\Lambda_c \, k z_c \, \epsilon^{\frac{1}{2}}}$
- Wave forcing: $T_w = \frac{1}{f_w}$



- Slow waves: $c/u_* < 13$
  - turbulent fast compared to critical layer wrap up
  - Critical layer wraps up with forcing from few crests
- Fast waves: $c/u_* > 13$
  - turbulence slow compared to critical layer wrap up
  - Critical layer high and so wave forcing weak

Conclusions?

- Physical picture to critical layer mechanism
- Critical layer is unsteady problem
- Leads to pulse of momentum
- Mechanism robust to complex wave forcings
  - Wave groups
  - Breaking waves
- But
  - Does mechanism persist with 3d turbulence?