Caroline Mary Series: Pearl of Hyperbolic Manifolds >>>

Caroline Series

Interview of Caroline Mary Series by Y.K. Leong

In each and every dust mote are infinite bodies
With cloudlike transformations pervading everywhere …
-- Avatamsaka Sutra (The Flower Adornment Sutra)
Volume 7, Book 4 (B.T.T.S.)

Caroline Mary Series has made important contributions to the theory of Kleinian groups, hyperbolic 3-manifolds and dynamical systems.

Educated in Oxford University, Series won a Kennedy Scholarship that took her to Harvard University where she wrote her PhD thesis on "Ergodic actions of product groups" under the supervision of George W. Mackey. Immediately after that, she did short stints at the University of California, Berkeley and Newnham College, Cambridge, and then moved to University of Warwick in 1978, became full professor in 1992 and went on to establish a distinguished career until the present time. In July 2011, a workshop on *Aspects of Hyperbolicity in Geometry, Topology and Dynamics* was held in University of Warwick to celebrate her 60th birthday.

Series was awarded the Junior Whitehead Prize of the London Mathematical Society in 1987. She has been invited to deliver lectures at numerous conferences, workshops and important meetings in the UK, Europe, US, Japan, New Zealand and Singapore; among them the International Congress of Mathematicians, a Royal Society Meeting on Chaos and the British Mathematical Colloquium. She has also actively popularized mathematics and promoted mathematical awareness in the UK by giving many public lectures to school students, undergraduates and the general public. She was featured several times on Radio 4 and in the BBC documentary *The True Geometry of Nature*. She was President of the Mathematical Sciences Section of the British Science Association in 2011.

In addition to making deep research contributions, she has organized many conferences and rendered professional and advisory service to the London Mathematical Society, the European Mathematical Society, the Isaac Newton Institute, the British Science Association, RAE 2008, Oxford Mathematics Institute, EPSRC Mathematics College and the Association of Commonwealth Universities. She is an editor of the journals *Mathematical Research Letters* and *Conformal Geometry and Dynamics*.

She is a co-author of the famous book *Indra's Pearls: The Vision of Felix Klein* published in 2002 after a 10-year protracted collaboration with David Mumford (Fields Medal 1974) and David Wright (and the computer). It is one of the few books which lead one through the labyrinths of hard mathematics into the inner visual world of fractals associated to 3-dimensional hyperbolic geometry.

Series is well-known for her relentless and tireless efforts in promoting the professional interests of women mathematicians around the world. In particular, she is a founder member of the organisation European Women in Mathematics in 1986 and is currently Chair of the European Mathematics Society's Women in Mathematics Committee.

Well before the Intergovernmental Panel on Climate Change sounded the alarm on global warming in its by-now famous report in 2007, Series started an initiative for the formation of an environment committee in the University of Warwick in 2000, followed by a climate change seminar and related activities. She continues to be on the University Environment and Amenities Committee.

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She was co-chair of the IMS (Institute for Mathematical Sciences) program on *Geometry, topology and dynamics on character varieties* (18 June-15 August 2010), which was sponsored by Global COE (Center of Excellence) of the Tokyo Institute of Technology, Compview and the National Science Foundation (USA). During her visit to the Institute, Y.K. Leong interviewed her on 12 August 2010 on behalf of *Imprints*. The following is an edited transcript of the interview in which she talks about her passion for mathematics, her fascination with the beautiful and tantalizing world of fractals and her personal commitment to women mathematicians and to the environment.

Imprints: At which point in your education did you decide to choose mathematics as a career?

Caroline Series: When I was about 14 years old and we had recently studied geometry in high school, we were given a problem for homework. I spent the whole evening trying to solve it. I succeeded and later I discovered that nobody else in the class had done it. From that time I resolved that I would always try to solve every problem which we were given in mathematics. I had a real passion for mathematics. My ambition then was to go to university and study mathematics as an undergraduate. Beyond that life ended and I never thought about it anymore until I was about to graduate from Oxford University. Then I wondered what to do next. I thought that maybe I would be a schoolteacher. But almost accidentally and not knowing what else to do, I was influenced by a friend to try for a master's in the US and perhaps go on to a PhD. Until I had finished my PhD I was uncertain about having a career as an academic mathematician.

I: You obtained your BA in mathematics at Oxford University but you did your doctorate at Harvard University. Wouldn't it have been expected for you to continue your postgraduate studies at Oxford? Was there any reason for not doing so?

S: Actually I was born and grew up in Oxford. My father [George William Series, FRS (1920-1994)] was a physicist in Oxford University and I had always intended that as an undergraduate I would move to Cambridge. However for various reasons I ended up studying in Oxford. After that I thought it was time I should change and I had a close friend who was recommended to go to Harvard to study for a PhD. So I decided to try to go to the US to study. It was more or less accidental. For my career it was an extremely important choice and I have recommended it to several students since. I just recommended a good undergraduate student to go to the University of Illinois in Chicago. I think it is very good to have international experience.

I: Were you on a scholarship at Harvard?

S: Yes. I won a very nice scholarship for the first two years, called the Kennedy Scholarship. It was a fund set up in memory of President Kennedy to allow some British students to go to either Harvard or MIT. After a couple of years, I was given a teaching assistant position. By the end of my PhD, I had no money left.

I: How did you choose your supervisor at Harvard?

S: When I arrived in Harvard, I hadn't really done enough research on the type of mathematics done there. I had imagined that because it was in the US, the graduate school would be enormous, but the mathematics department was actually quite small. Most of the professors were doing either algebraic geometry or number theory and I was more interested in analysis at that time. So there were really not many professors to choose from. One was about to retire and another I felt too intimidating. I chose George Mackey because I thought he was more geometrical, more analytical than others.

I: I notice that the topic of your PhD thesis is something about ergodic theory. Isn't it a bit probabilistic?

S: Yes, it is a bit probabilistic. George Mackey was, of course, a great expert on representation theory of Lie groups. But at that time, he was very interested in groups acting on measure spaces because this gives good examples of group representations. He had rather interesting and very original ideas. His other students at that time, including Bob Zimmer, worked on his ideas about what he called virtual groups, which turned out to have relations with many other topics. Also, early on I was very uncertain if I could complete my PhD. I thought that I could perhaps instead get a master's degree in statistics and become a statistician. I didn't quite do that, but I did take some statistics course. Studying ergodic theory fit in and would have made it easier to switch.

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I: You taught at University of California for only one year immediately after your PhD before you returned to England to pursue and develop your academic career. Was this due to some kind of cultural pull?

S: I always had the intention of returning to the UK in the long term. After I finished my PhD I was in a fortunate position. I was offered a temporary lectureship in Berkeley which academically was my first choice. But I was also offered a position in Newnham College, a women's college in Cambridge (UK). It was a research fellowship but it was pretty certain it would become a permanent college teaching job. I thought I couldn't give up the opportunity of a likely permanent job in a Cambridge college. So it was agreed that I would go to Berkeley for one year and then go back to England. In fact, I only stayed in Cambridge for one year. At that time there were very few people there who had related mathematical interests. I was very lucky I got a job in Warwick where there was a big group in ergodic theory.

I: If I'm not mistaken, Cambridge was rather male dominated at that time.

S: At that time, it was exceedingly [male oriented], socially and academically. But I did make a few good acquaintances, particularly S. J. Patterson; we found our work had an unexpected scientific overlap and our connection has continued over the years.

I: According to David Mumford, the book *Indra's Pearls: the Vision of Felix Klein* which you wrote with him and David Wright took 20 years in the making and writing. Mumford and Wright were in the US while you were in England. How did this collaboration come about?

S: Of course, I knew Mumford when I was a student in Harvard, but this has nothing to do with our book. What happened was that some years after I left, Mumford became interested in making computer generated pictures of limit sets of Kleinian groups, which are rather like Julia sets. David Wright was at that time a graduate student in Harvard, and very interested in computing. Together they made a large number of astonishingly beautiful pictures. They wanted to create an elegant coffee table type book of the pictures but it never took off because every time they met they would spend the time creating more pictures. I saw some of their pictures and they fascinated me. I began trying to prove some facts about them. Mumford came across a popular article I had written about the pictures and he asked me to join them writing the book. It took a long time, a full 10 years, but we finally succeeded. It was difficult partly because we didn't very often have the opportunity to meet. One of us would write a long piece and send it to the others. The others, of course, wouldn't answer because they were busy with other things. Finally we would meet and decide we didn't like what had been done and we would start all over again. This went on for years. It was really our publisher, David Tranah at Cambridge University Press, who had faith in the project and wanted it to succeed. He began to give us more and more pressing deadlines and finally the book took shape. Without him it would never have been written.

I: The patience paid off.

S: Yes, I'm very happy with it. David Tranah gave us constant encouragement and support.

I: I must say that the pictures are really very nice. How did you do the colours?

S: David Wright did the colours. It's not hard to add colours; for example, you are plotting things at different levels, level n, level n+1, and so on, and you can cycle the colours. David Wright enjoyed playing with it. Since then some people have taken it up as a much more artistic enterprise, particularly Jos Leys who is a wonderful mathematical graphics artist. He modified our programs to create very beautiful pictures; one has to pay attention to colour, arrangement and background. It's a great skill.

I: The book *Indra's Pearls* is largely computational in approach. Does this represent an alternative, if not new, approach to research in geometry? Has this approach contributed significantly to research in geometry?

S: It's not just our book. Computational pictures have made very important contributions to geometry in recent years. The whole theory of complex dynamics has been inspired by the computational aspect. The pictures Mumford originally liked actually had a lot of influence because when you see them, you understand that something that conjecturally occurs really exists. You begin to see intricate structures and this inspires people to go and try to prove things. It's a two-way thing. The computer generated pictures have inspired a lot

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of research and then the research suggests we make further pictures. They go hand-in-hand.

I: Did any of those pictures suggest new theorems?

S: Yes. They suggested theorems of mine. I believe they also contributed to a very important result proved by Yair N. Minsky called the Ending Lamination Conjecture. Perhaps you have a theorem in mind but seeing the pictures makes you feel more certain there must be something in it and you become more determined to prove it.

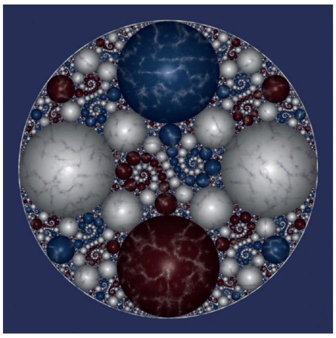
I: Did it suggest any counterexamples?

S: I think it showed unexpected phenomena. You see something that you don't understand fully and you realize you have to pay attention. The pictures in our book are 2-dimensional. Eventually I learn that in order to prove results about them one needs to use 3-dimensional hyperbolic geometry. It took me a long time to understand this. The 3-dimensional geometry is very important.

I: It must be hard to imagine 3-dimensional space.

S: It's hard to draw pictures in 3-dimensional space. Now I'm looking at parameter spaces in higher dimensions. One difficulty is how to present computations; all you can do is to show two dimensional slices and really you want to see the full picture.

I: In the *Avatamsaka Sutra*, which you quoted in several places in your book, there is mention of an infinite web of pearls which mutually reflect each other and all the reflections in each pearl. This is an amazing analogy with a religious world view of the mutual interconnectedness and dependence of all things. Do you see any parallels in such viewpoints and your own mathematical viewpoints? For example, do you have a Platonist viewpoint that fractals exist "out there" and not just as mental constructs.



Kleinian Groups Orbit Trap 01
(Courtesy Ross Hilbert)

S: The last part is a very fascinating question. Mathematics exists in our minds and yet it gives us such a powerful way of dealing with the external world. One reason I like working with these computer generated pictures is that you start with an abstract mathematical construct but as you plot it by computer the reality seems to grow. All the things that you have proved you see vividly in front of your eyes. And somehow one can't argue with the pictures; they are so concrete. As for fractals, I find it astonishing that so many natural objects have outlines or structures that are fractal-like. It seems to me that the concept of fractals captures something very important in nature. I imagine that fractal geometry and its applications will become more and more important as time goes on.

I'm not sure about the religious part. But one thing that strikes me is that different cultures have different concepts of infinity. In the western tradition, we think of infinity as counting: 1, 2, 3, 4, going on forever in a linear way. The concept of infinity in Buddhist and Hindu writings is much more to do with objects subdividing into parts and then each part subdividing again and again forever. This leads to an uncountable infinity of limit parts. It's a very different way of thinking. I think it must really colour the world view of these cultures. Western culture is very goal-oriented and likes to measure everything whereas the Eastern culture is more about inner structures and realities. Another example is the Maya in South America – their calendar worked in cycles. That is another idea of infinity. I suspect these different viewpoints profoundly affect people's attitudes to life. If I were more of a philosopher or an ethnologist, I would like to study this more.

I: Can I ask you if you have read the *Avatamsuka Sutra*?

S: I can't say I have read the whole thing. I bought a translation (it's translated into English now) and I did read a considerable part of it while we were writing the book. I was always looking for suitable quotations. The book is extremely long. One thing is amazing. At various points are descriptions of large numbers of things: buddhas, jewels, worlds within worlds, and so on. The numbers described are huge, gigantic numbers. The people who were writing this ancient text must have had the concept that infinity was truly unimaginably huge. (Of course, I am speaking here about what I found in the English translation, not the Sanskrit or Chinese.)

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I: Now that Thurston's Geometrization Conjecture has been recently solved by Perelman [Grigori Perelman (Fields Medal 2006)] and others, is it fair to say that everything of importance is now known about hyperbolic 3-dimensional manifolds?

S: It is true that the Geometrization Conjecture has been solved by Perelman and others. But actually there has been another extremely important development that happened more or less at the same time, done by Yair N. Minsky and his co-workers. This is the Ending Lamination Theorem I mentioned above. They managed to carry out a program initiated by Thurston [William Thurston (1946-2012), Fields Medal 1982] to classify infinite volume hyperbolic 3-manifolds using some rather simple invariants associated to their ends. From the knowledge of the end invariants, they show it is possible to reconstruct the entire geometry of the manifold. This is transforming the subject. Previously people were always trying to understand whether there could be any "wild" hyperbolic manifolds lurking about somewhere. Now we know that in a precise sense they are all "tame". We know how to handle all the infinite geometry. In that sense, it is true that the big questions were solved within the recent past. I think it is true the field has changed since this happened. But there are still many things to do, which one can now tackle using the manifold structure. I'm currently thinking about how limit sets evolve as the manifold is deformed. Minsky's theory is just what is needed. People in the hyperbolic geometry community are also now looking at different kinds of geometry and geometric structures, sometimes related to physics. Our current program at NUS brings together different kinds of geometry in a broad way.

I: Is it possible for one single person to read through all those proofs of the classification results? (I'm thinking of the situation in the classification of finite simple groups.)

S: It's not bad as that. There is a group of people who understand in principle how it [the proof] works. I can't say that I have read it all. I think it is perhaps more conceptual. There are some very important geometrical and topological principles, and once you understand the principle and the outline structure of the proof then you can go and check the details. It is a reasonable thing to hold in your mind. I doubt that there are many people who have gone through everything but it is feasible. There have been some simplifications and no doubt it will be cleaned up further – it's a matter of time.

I: Is the next step in geometric research on manifolds to go on to dimension 4?

S: There is a community of people who work on dimension 4, but it has a quite different flavour. For hyperbolic geometers it's not so much dimension 4 but investigating different kinds of geometric structures. This could be in dimension 4, but it could be much broader.

I: You have been very active in promoting and supporting opportunities for women in pursuing careers in mathematics for many years. What have been the achievements of the organization "European Women in Mathematics" [EWM] of which you are a founder member?

S: When we set this up, it was not really to be a campaigning organization. Rather, we wanted to create a network of women to exchange ideas, share problems, encourage each other, meet at conferences and so on. I believe we have had some influence. It was set up almost 30 years ago now, and there have been a considerable number of programs in different countries and different institutions to encourage and give more recognition to women. This could be inviting women to give prestigious lectures, or special meetings or grants. Some universities have made it a point to make sure that they have women on committees. Particularly in Germanic countries, it used to be very difficult for women, but now the climate has become quite favourable. We were a group of women from many different countries and this gave us a wide perspective. We gathered some statistics and it was very noticeable that there are a lot more women mathematicians in some countries than others; it is partly a cultural issue. I think we probably encouraged a number of young women who without us would have taken up some other job. Women usually have to carry the burden of family life. We had some successful older members who could say, "Well, it's difficult but I've managed to succeed and this is how I did it." This has been encouraging to other women.

I: Is this organization centred in England?

S: No, it is pan-European. There was already a big organization in the United States. Our idea was that we should have a counterpart in Europe. We have a main meeting once every two years somewhere in Europe, which attracts 70 to 80 people. We also have a number of members from outside Europe. Our structure is set up so that women

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anywhere in the world can come along and say they want to establish a group in their country. Just before the ICM [International Congress of Mathematicians, 19 – 27 August 2010, Hyderabad] there's going to be a meeting which was initiated by EWM and which is being organized in collaboration with a group of Indian women. We hope there will be a group set up in India and perhaps other countries. There is a group in Korea and Japan now. Our structure is very flexible and informal.

I: Could you tell us something about your interest and involvement in activities concerning environmental issues and climate change?

S: This is something I have become increasingly concerned about in recent years. It began when I became involved in our university's recycling. We were not recycling anything very successfully. The system was reliant on charitable and voluntary actions, and it didn't really work. Eventually I came to realize that you needed something in the infrastructure of the University to take care of environmental issues. It's no good just relying on people's goodwill. So around 2000 I managed to set up a committee in the university. I made contact with a professor from the Business School and together we approached the university administration. We set up an informal group of some academics and some administrators. For several years we were not very successful because we had no proper resources. Finally, the university decided to hire an environmental officer whose job was to be a focal point for environmental issues within the university. He was responsible for handling issues like recycling, energy consumption, water consumption, and transport problems. Now we have an official university committee with an environmental officer, transport officer, recycling officer, energy officer, and the university is making very substantial efforts to reduce its carbon footprint. We have a cost effective recycling scheme. I'm very happy with it. I was right; such things have to be integrated into the university infrastructure.

On a personal level, about 10 years ago I put hot water solar panels on my house. Recently, our government introduced a subsidy if you generate electricity using solar electric (PV) panels. The panels are quite expensive to put it up, but they generate quite a lot of electricity even in the UK. You get quite a lot of money for the electricity you generate, so the panels will pay for themselves in about 10 years. I put up PV panels and it's like a long-term investment. In the UK environment groups were trying to convince the government to do this for many, many years, because previous policies were not working very successfully. The last Labour government made a lot of glamorous commitments but didn't actually do very much practical. But in the last couple of years, things have changed because the European Union has become quite serious about reducing its carbon footprint. It's now becoming almost a legal requirement and serious money is involved. There is recycling in the UK now instead of disposing rubbish by burying it underground. The practical cost of environmental actions can be high and until serious economic incentives are brought in so that it makes economic sense, they don't really happen in a mainstream way.

I: I remember hearing over BBC one year ago that some company in England was trying to recycle plastics. I thought that was a good idea. Do you know anything about it?

S: Oh, yes, there's a lot of recycling of plastics. In fact, our university collects not just bottles, but all kinds of plastics, mixed plastics. For example, people wear a lot of fleece jackets in colder countries, and a lot of them are made of recycled plastics. Recycled plastic is used to make insulation materials. People are working on developing the technology to make the recycling process less difficult (there are many different kinds of plastics) so that you don't have to separate different kinds of plastics. Plastics are everywhere. They do not biodegrade and last thousands of years. We need to find better ways to deal with them and I'm sure we will.

I: What advice would you give to students, especially women students, who would like to pursue a research career in mathematics?

S: My advice is that first of all, you must really like doing mathematics. It may not be a glamorous career. But if you really want to do it and love mathematics, and you are persistent, then you will find a way to succeed. What I did and what worked for me is, at the beginning of your career try to go to the best places you can and meet the leading people in your field. That way you get a good grounding. Later on, when you have to find a permanent job and you perhaps have family constraints, then you have got a good basis and good contacts, so wherever you settle, you are able to continue research. For me it has been a wonderful career and I would recommend it.